Transverse Spin Physics

Lecture II

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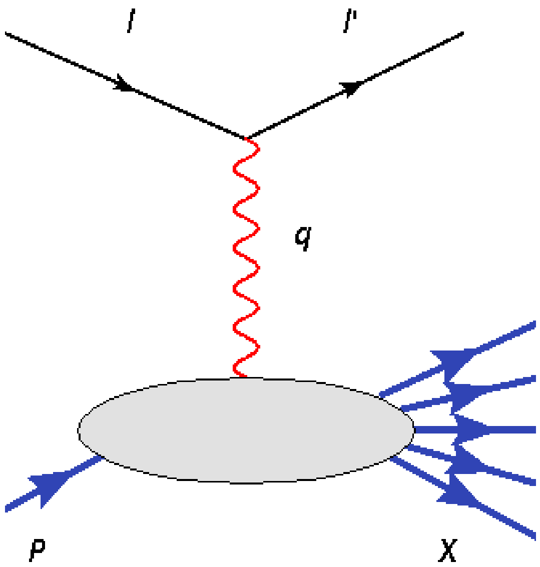
The plan:

- **Lecture I:**
  Transverse spin structure of the nucleon
  Overview of past experiments
  History of interpretation
  Overview of present understanding
- **Lecture II**
  Transverse Momentum Dependent distributions (TMDs)
  Sivers function
  Twist-3
- **Lecture III**
  Transversity
  Collins Fragmentation Function
  Global analysis
- **Lecture IV**
  Evolution of TMDs

Transverse Momentum Dependent distributions

Deep Inelastic Scattering (DIS)

In order to access **distributions** we could use
deep inelastic scattering



The energy is big enough to transform the proton in a lot of final states

Bjorken limit is

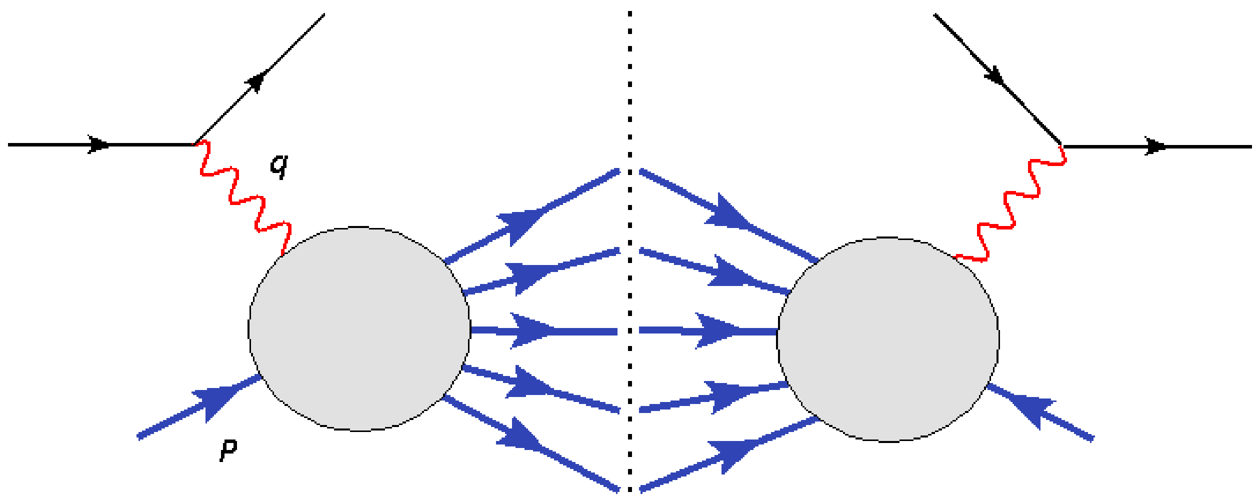
$$\mathbf{Q^2 \to \infty}$$

$$\mathbf{P \cdot q \to \infty}$$

$$\mathbf{x_{Bj} \equiv \frac{Q^2}{2P \cdot q} \to const}$$

Deep Inelastic Scattering (DIS)

Distributions measured in deep inelastic scattering

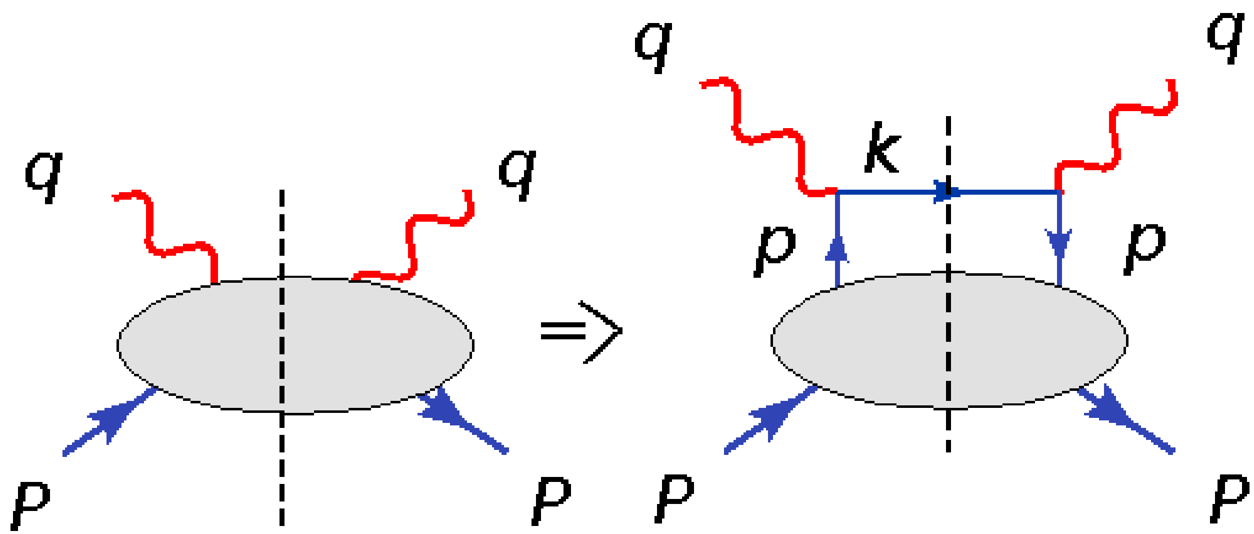


$$\sum_{X} |X\rangle\langle X|$$

This sum makes it sensitive to parton structure!

Distributions and parton model

Parton model is a logical step, partons are pointlike and dilute, so photon interacts with them incoherently



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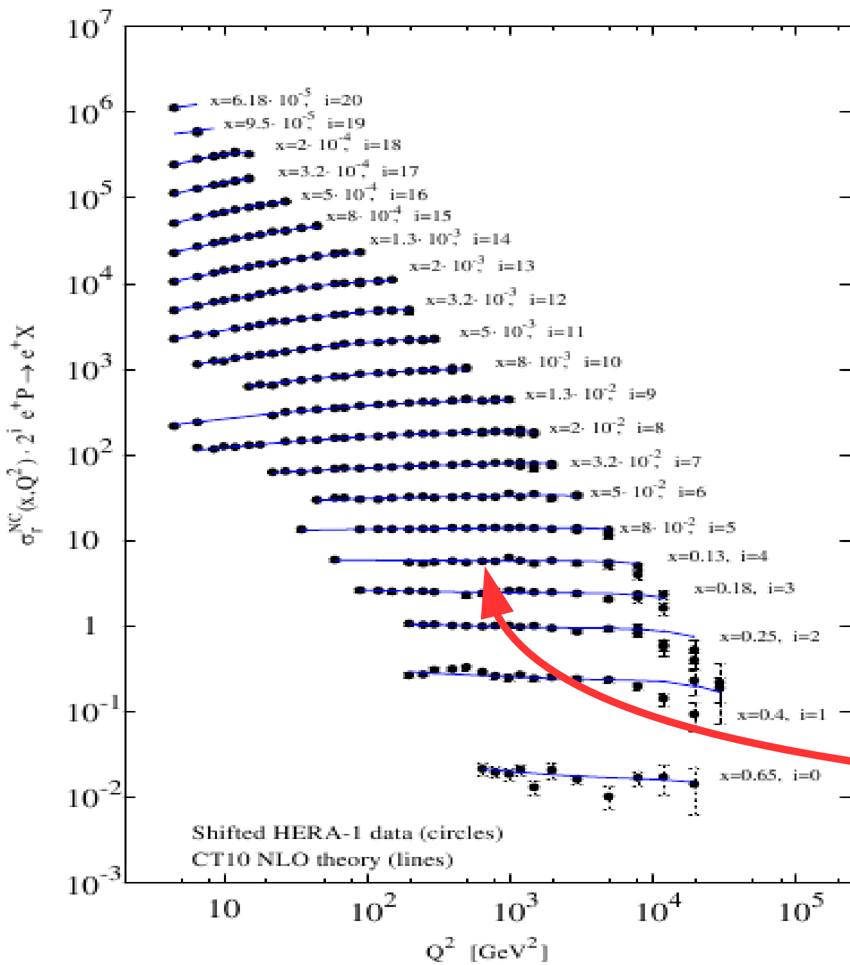
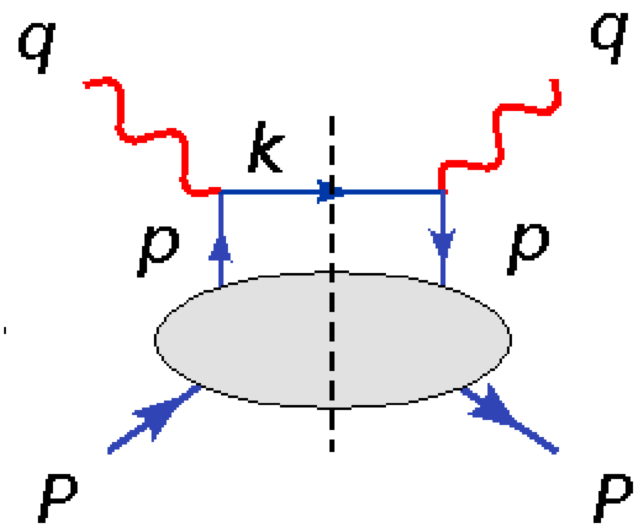
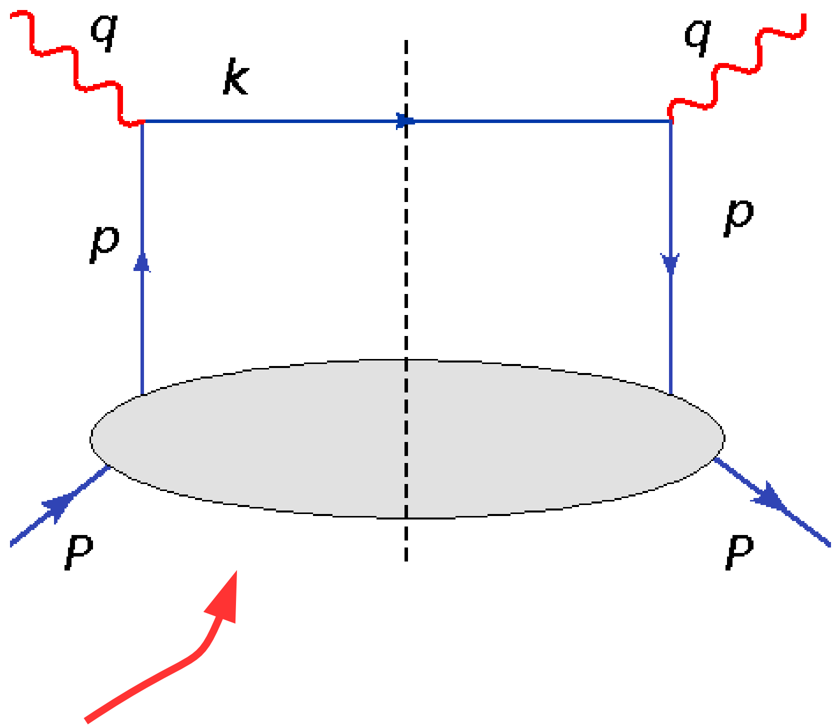




CONSTANT!

Distributions and parton model

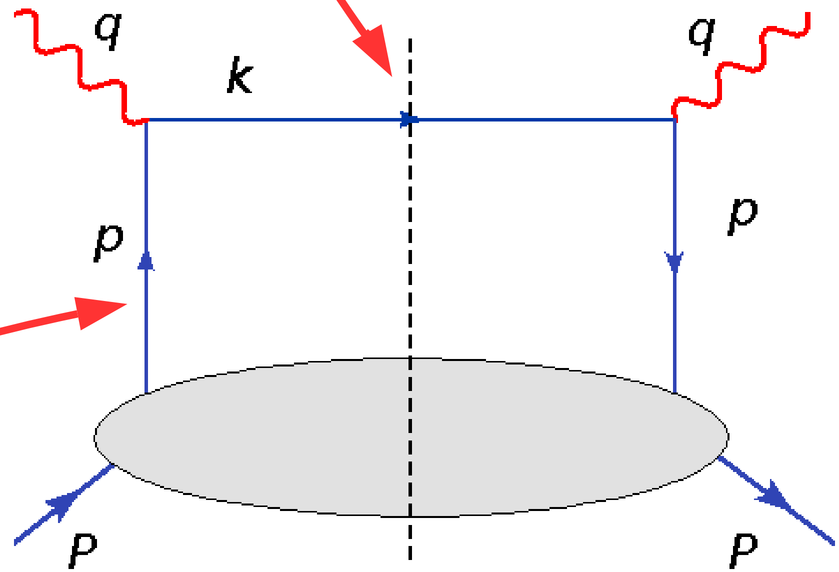
This diagram is called “handbag diagram”



$\mathbf{\Phi(p, P)}$ - parton distribution

Why quarks are on mass-shell?

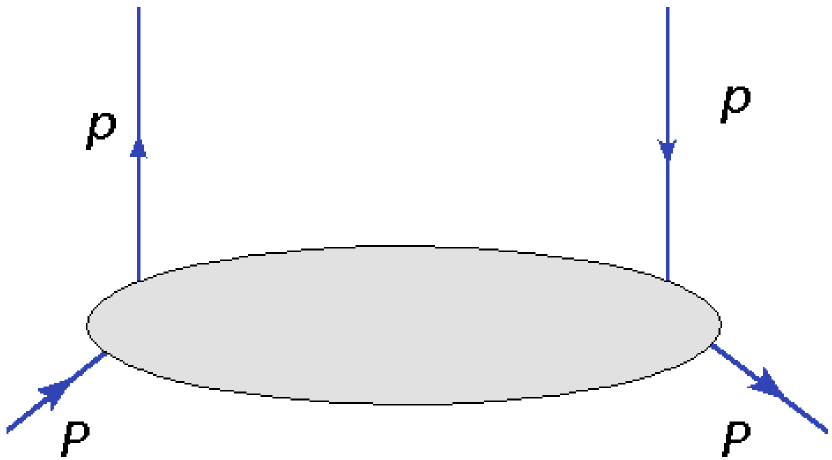
$$Im\left(\frac{1}{k^2+i\epsilon}\right)=\pi\delta(k^2)\qquad\Rightarrow\quad k^2\approx 0$$



This one is virtual! However the main contribution comes from

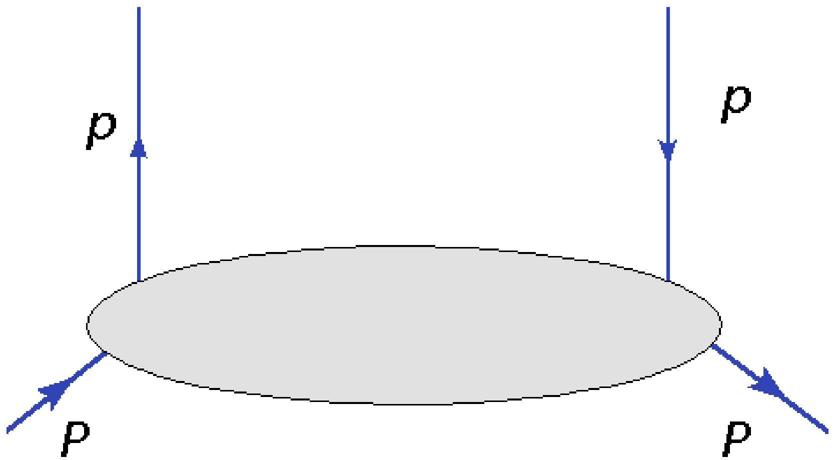
$$\int d^4p\left(\frac{1}{p^2+i\epsilon}\right)\left(\frac{1}{p^2-i\epsilon}\right)\quad\Rightarrow\quad p^2\approx 0$$

Definition of parton distribution



$$\Phi_{ij}(p,P) = \int \frac{d\xi^+ d\xi^- d^2\xi_T}{(2\pi)^4}\, e^{ip\cdot\xi}\, \langle P, S_P | \bar{\psi}_j(0)\psi_i(\xi) | P, S_P \rangle$$

Definition of parton distribution



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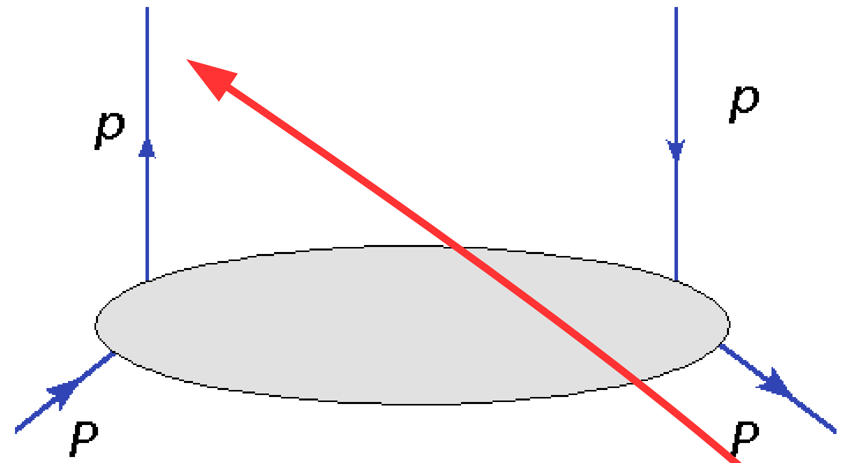
Fourier transform from coordinate to momentum space

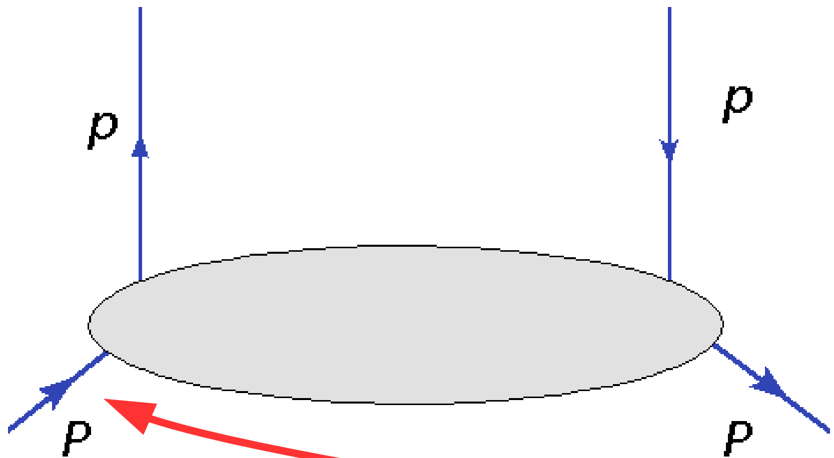
Definition of parton distribution



$$\Phi_{ij}(p,P) = \int \frac{d\xi^+ d\xi^- d^2\xi_T}{(2\pi)^4}\; e^{ip\cdot\xi}\; \langle P,S_P|\bar{\psi}_j(0)\psi_i(\xi)|P,S_P\rangle$$

Quark field operator

Definition of parton distribution



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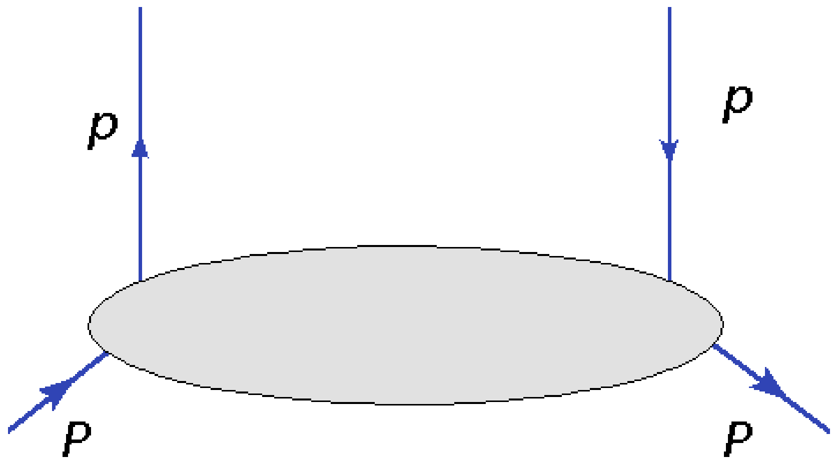
The proton state vector

Definition of parton distribution

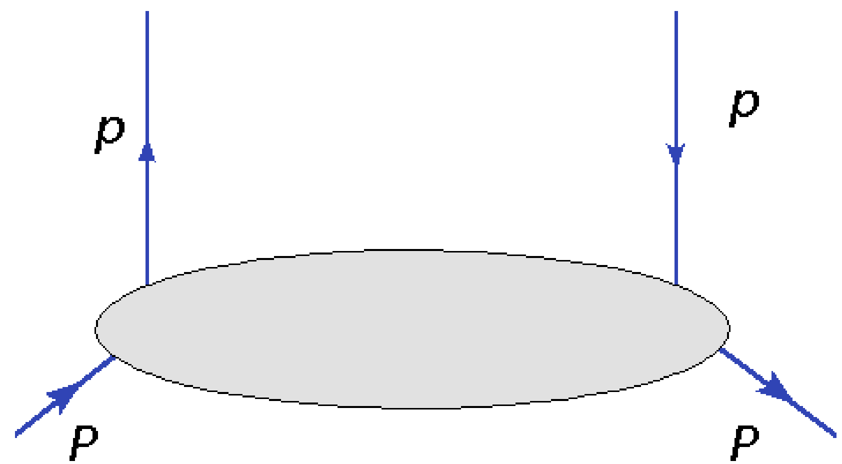


$$\Phi_{ij}(p,P) = \int \frac{d\xi^+ d\xi^- d^2\xi_T}{(2\pi)^4}\; e^{ip\cdot\xi}\; \langle P, S_P|\bar{\psi}_j(0)\psi_i(\xi)|P, S_P\rangle$$

Position of the field in coordinate space

Distributions and parton model

Definition of parton distribution



$$\Phi_{ij}(p,P) = \int \frac{d\xi^+ d\xi^- d^2\xi_T}{(2\pi)^4}\, e^{ip\cdot\xi}\, \underline{\langle P, S_P|\bar{\psi}_j(0)\psi_i(\xi)|P, S_P\rangle}$$

This matrix element is called "bilocal"

Distributions and parton model

What do we know about quark momentum? Suppose that proton is moving along Z direction with a high momentum, then

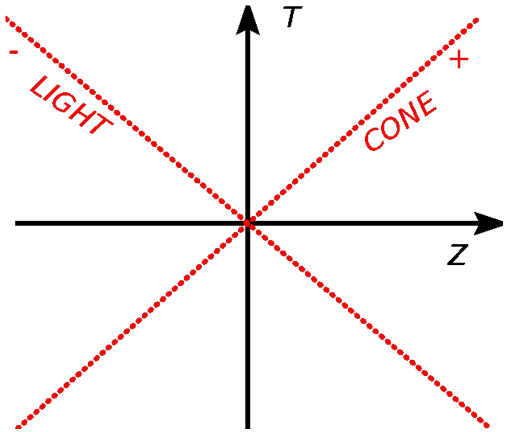
$$p^\mu = xP^+ n_+^\mu + \frac{p^2 + \mathbf{p}_\perp{}^2}{2xP^+} n_-^\mu + p_\perp^\mu$$

"Big"component $\sim Q$

$x = p^+/P^+$ is a new variable called lightcone momentum fraction

$$P^+ = \frac{1}{\sqrt{2}}\left(P^0 + P^z\right)$$

$$P^- = \frac{1}{\sqrt{2}}\left(P^0 - P^z\right)$$



What do we know about quark momentum?

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"Big"component $\sim Q$

"Small" component $\sim 1/Q$

What do we know about quark momentum?

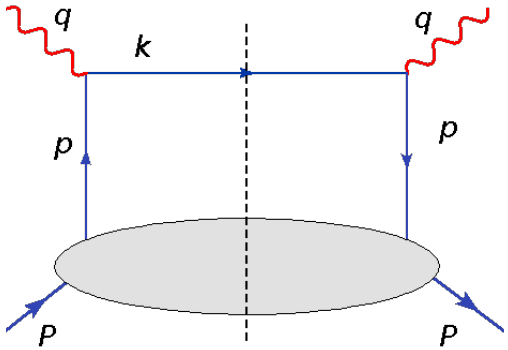
$$p^\mu = xP^+ n_+^\mu + \frac{p^2 + {\mathbf{p}_\perp}^2}{2xP^+} n_-^\mu + p_\perp^\mu$$

"Big" component $\sim Q$

"Small" component $\sim 1/Q$

"Transverse" component $\sim \Lambda_{QCD}$

What do we know about hadronic tensor?

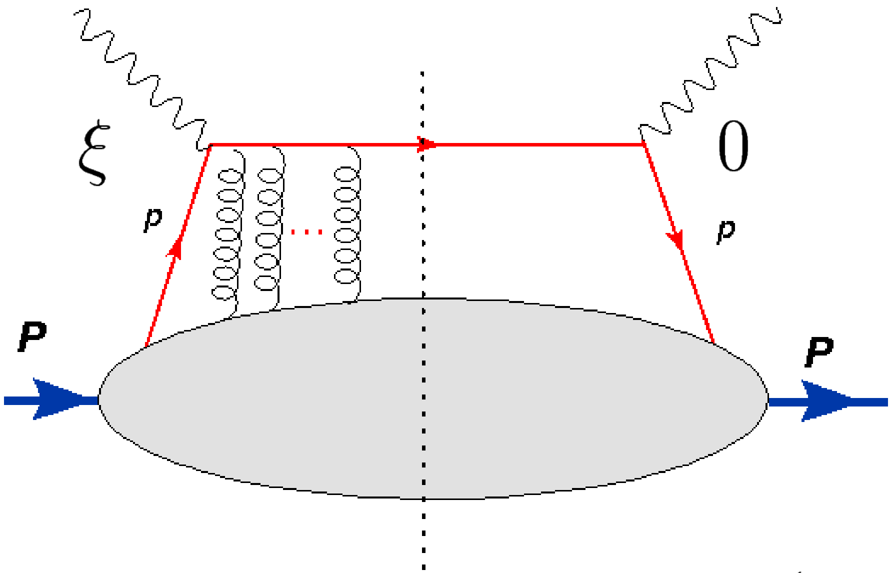


$$W^{\mu\nu} = \sum_q e_q^2 \int \frac{d^4p}{(2\pi)^4} Tr(\gamma^\mu(\not{p}+\not{q})\gamma^\nu \Phi(P,p))\delta((p+q)^2)$$

$$\delta((p+q)^2) \approx \delta(-Q^2 + 2xP\cdot q) = \frac{1}{2P\cdot q}\delta(x_{Bj} - x)\ ,$$

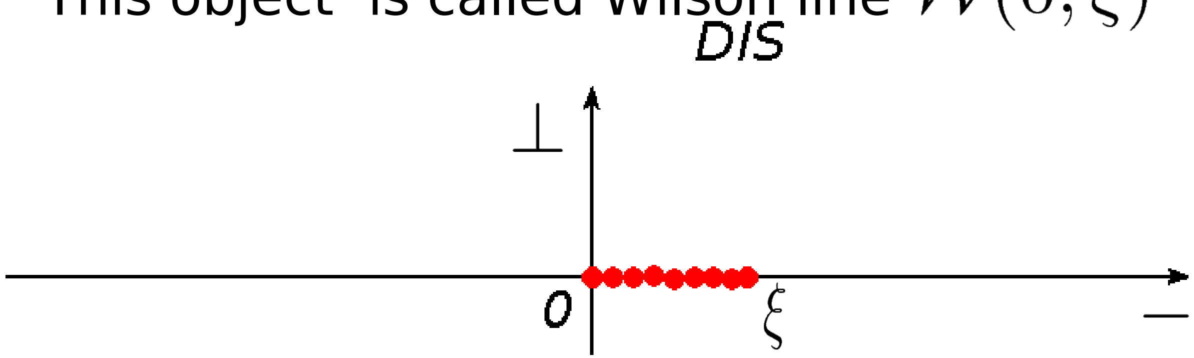
Quarks are "probed" at value of x_{Bj}

The quark and remnant are colored thus they interact via gluon exchanges!



This object is called Wilson line $\mathcal{W}(0,\xi)$

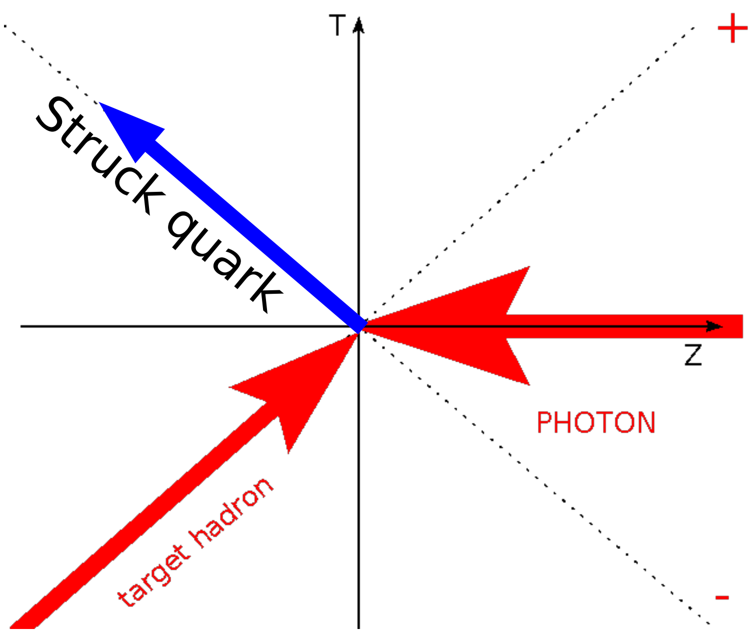
For DIS:



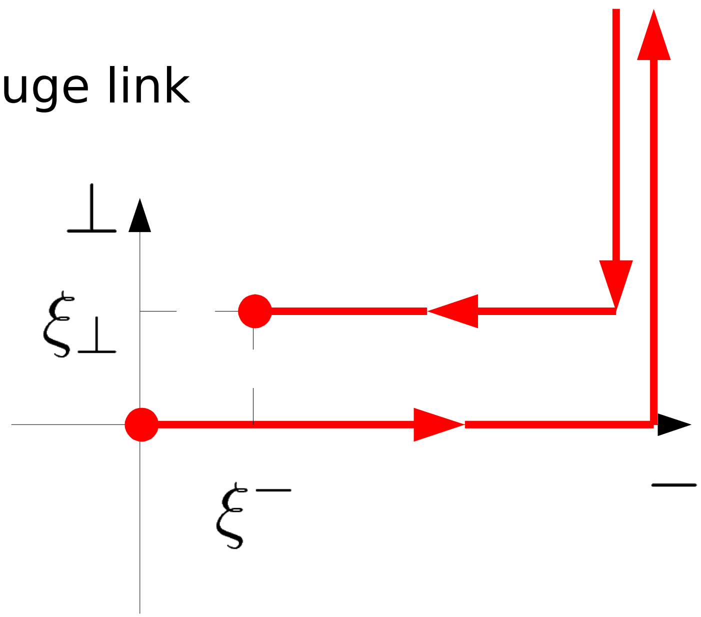
Transverse Momentum Dependent distributions

$$\Phi_{ij}(x,\mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ixP^+\xi^- - i\mathbf{k}_\perp \xi_\perp} \langle P, S_P | \bar{\psi}_j(0) \mathcal{U}(\mathbf{0},\xi) \psi_i(\xi) | P, S_P \rangle |_{\xi^+=0}$$

SIDIS in IMF:



Gauge link



$$\mathcal{U}(a,b;n) = e^{-ig\int_a^b d\lambda n\cdot A_\alpha(\lambda n)t_\alpha}$$

Ensures gauge invariance of the distribution, cannot be canceled by gauge choice

TMDs

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

8 functions in total (at leading Twist)

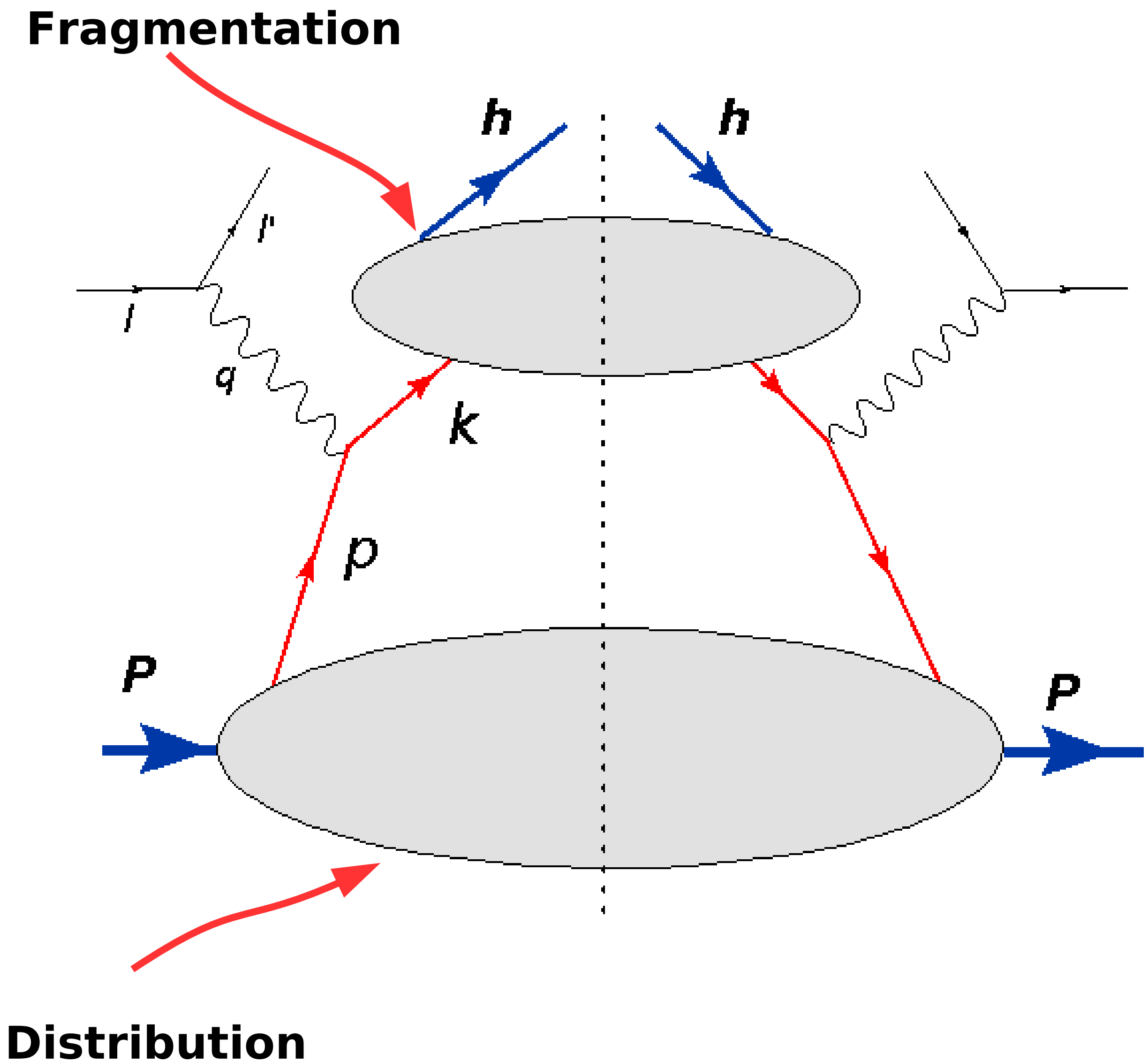
Each represents different aspects of partonic structure

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

Semi Inclusive Deep Inelastic Scattering (SIDIS)

Factorization



$$\sigma_{\color{red}{\mathbf{SIDIS}}}$$

$$||$$

$$\mathbf{D_{q/h}}$$

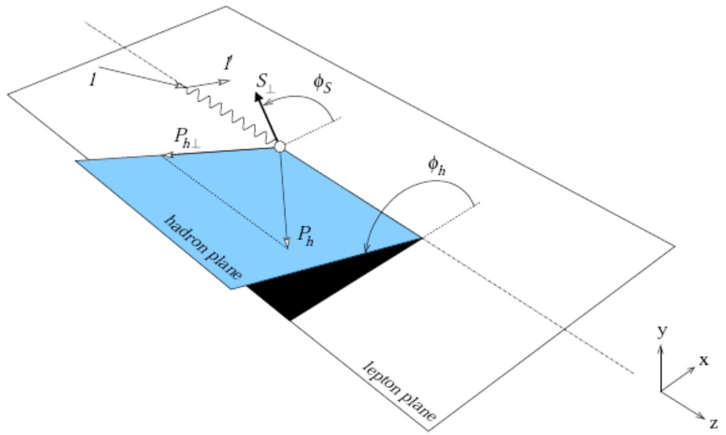
$$\otimes$$

$$\hat{\sigma}_{\mathbf{lq \to l'q'}}$$

$$\otimes$$

$$\mathbf{f_{q/P}}$$

Semi Inclusive Deep Inelastic scattering



One can rewrite the cross-section in terms of **18** structure functions

Each structure function encodes parton dynamics via convolutions of TMDs when factorization is applicable

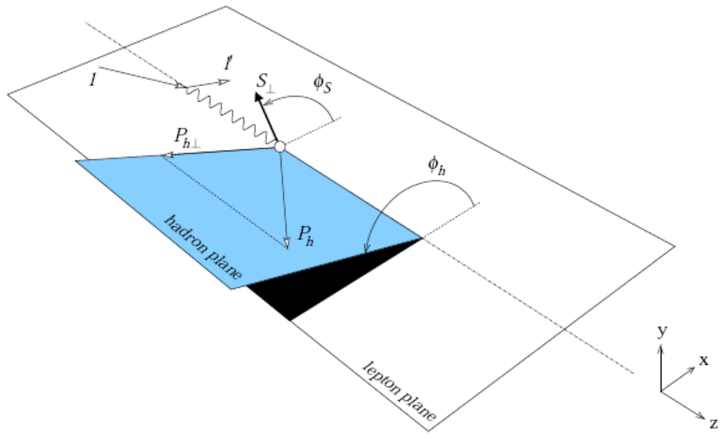
Mulders, Tangerman (1995),
Boer, Mulders (1998)
Bacchetta et al (2007)

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}\;+\ldots$$

Semi Inclusive Deep Inelastic scattering



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Mulders, Tangerman (1995),
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$$F_{UU,T} = x \sum_q e_q^2 \int \mathrm{d}^2 k_\perp \mathrm{d}^2 p_\perp \delta^{(2)}(\mathbf{P}_{h\perp} - z\mathbf{k}_\perp - \mathbf{p}_\perp) f^q(x, k_\perp^2) D_q(z, p_\perp^2)$$

TMDs

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

8 functions in total (at leading Twist)

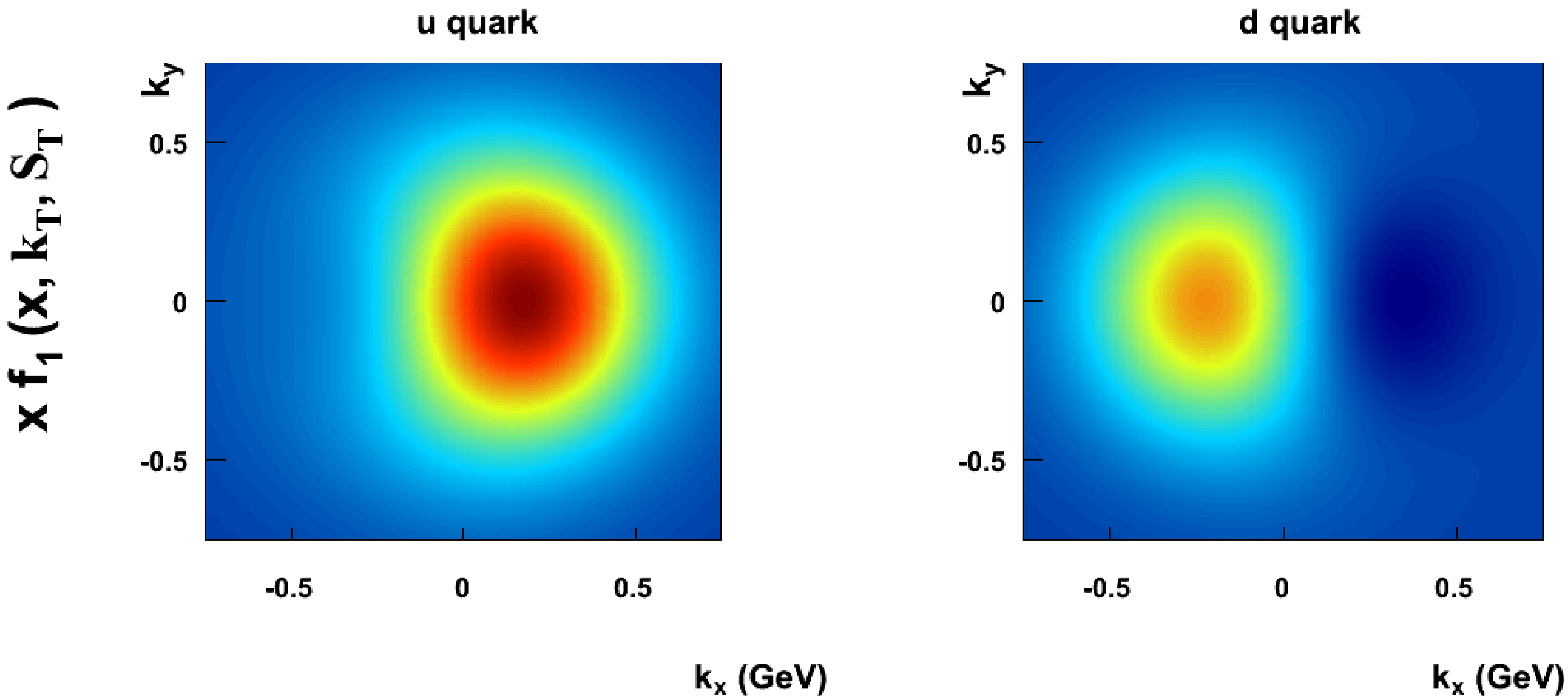
Each represents different aspects of partonic structure

Each function is to be studied

Sivers function

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

Tomographic scan of the nucleon



Anselmino et al 2009

Both proton and quarks are so-called spin-½ particles

Quarks are confined inside an extended proton and move – the motion creates Orbital Angular Momentum

Can this motion be correlated with the spin of the proton?

Yes, it can!

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\epsilon_T^{ij} \mathbf{k_{Ti}} \mathbf{S_{Tj}}}{M}$$

Yes, it can!

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Correlation of the spin and motion of the quarks

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Sivers function

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$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

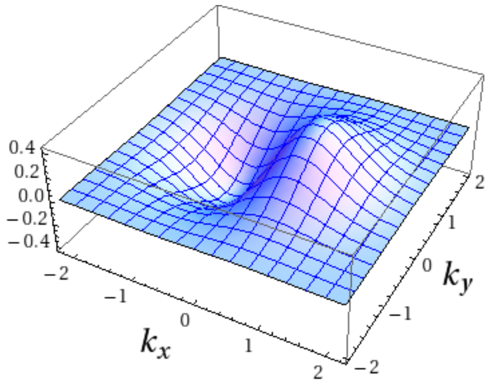
Suppose the spin is along Y direction:

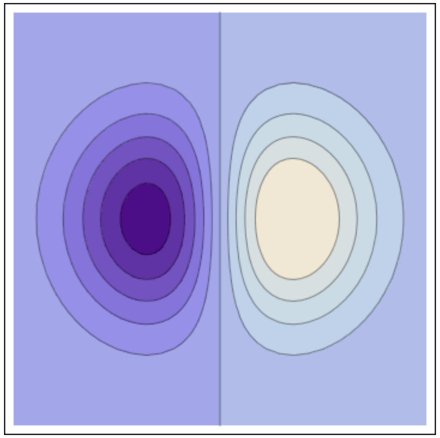
$$S_T = (0, 1)$$

Deformation in momentum space is:

$$x \cdot f(x^2 + y^2)$$

This is called "dipole" deformation.





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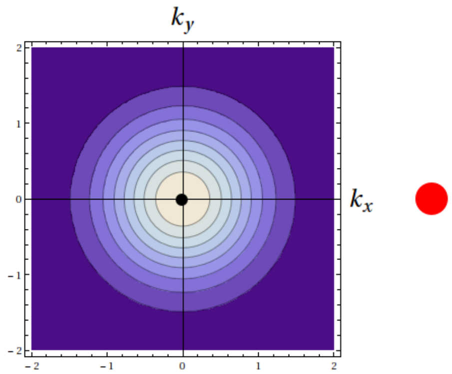
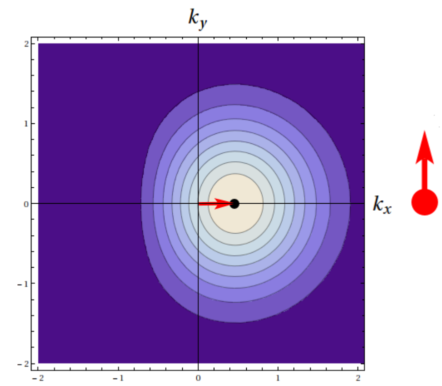
No correlation:

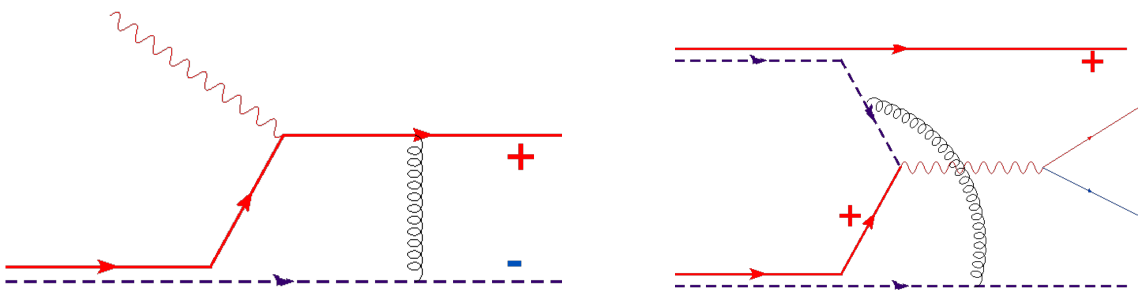


Correlation:



Sign change

Colored objects are surrounded by gluons, profound consequence of gauge invariance.
Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)



Brodsky,Hwang, Schmidt
Belitsky,Ji,Yuan
Collins
Boer,Mulders,Pijlman,
Kang, Qiu, AP
etc

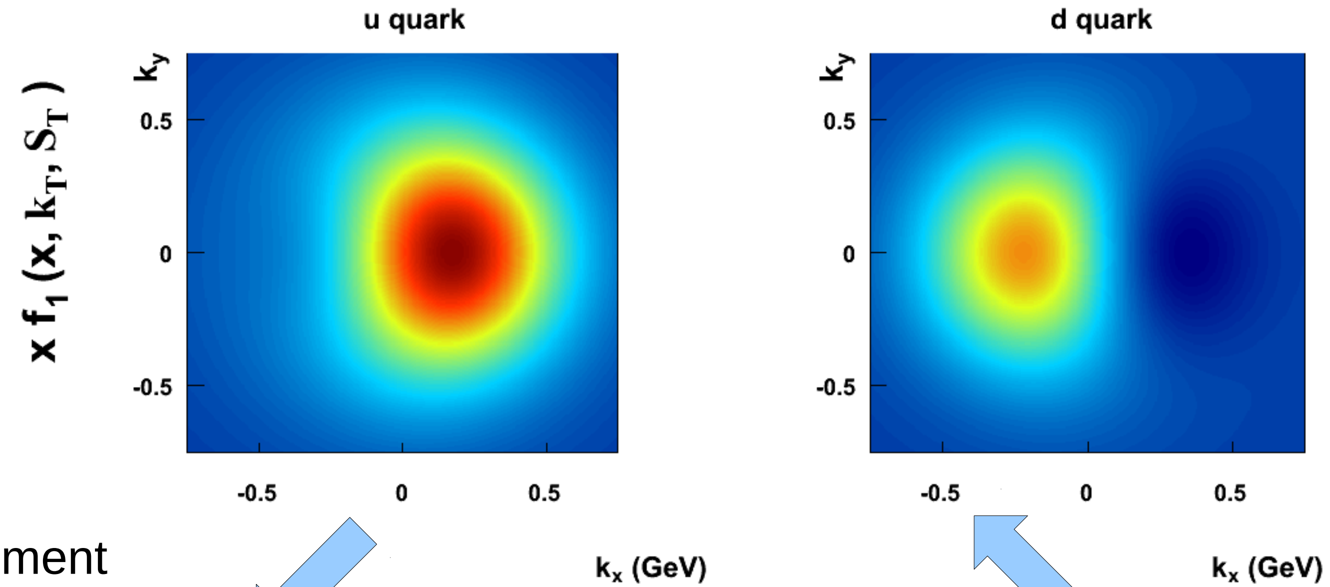
$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

One of the main goals is to verify this relation.
It goes beyond “just” check of TMD factorization.
Motivates Drell-Yan experiments
AnDY, COMPASS, JPARC, PAX, FERMILAB etc
Barone et al., Anselmino et al., Yuan,Vogelsang, Schlegel et al., Kang,Qiu, Metz,Zhou, AP etc

Global analysis

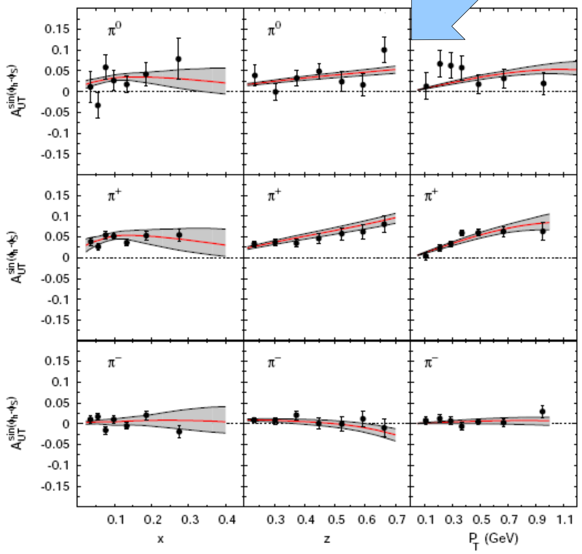
Tomographic scan of the nucleon

Theory



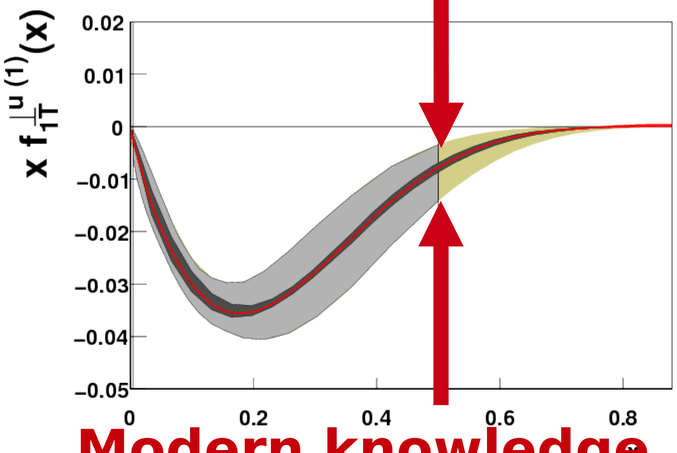
AP 2012

Measurement



Global analysis of the data

Results



Modern knowledge

Global analysis:

Model ansatz for TMDs,
Initial set of parameters

Evolve TMDs to relevant scale

Calculate observables and χ^2

Change parameters

No!

χ^2 minimum?

Yes!

Global analysis:

Model ansatz for TMDs,
Initial set of parameters

Parametrization must be flexible but economical in terms of number of parameters

Evolve TMDs to relevant scale

Calculate observables and χ^2

χ^2 minimum?

No!

Change parameters

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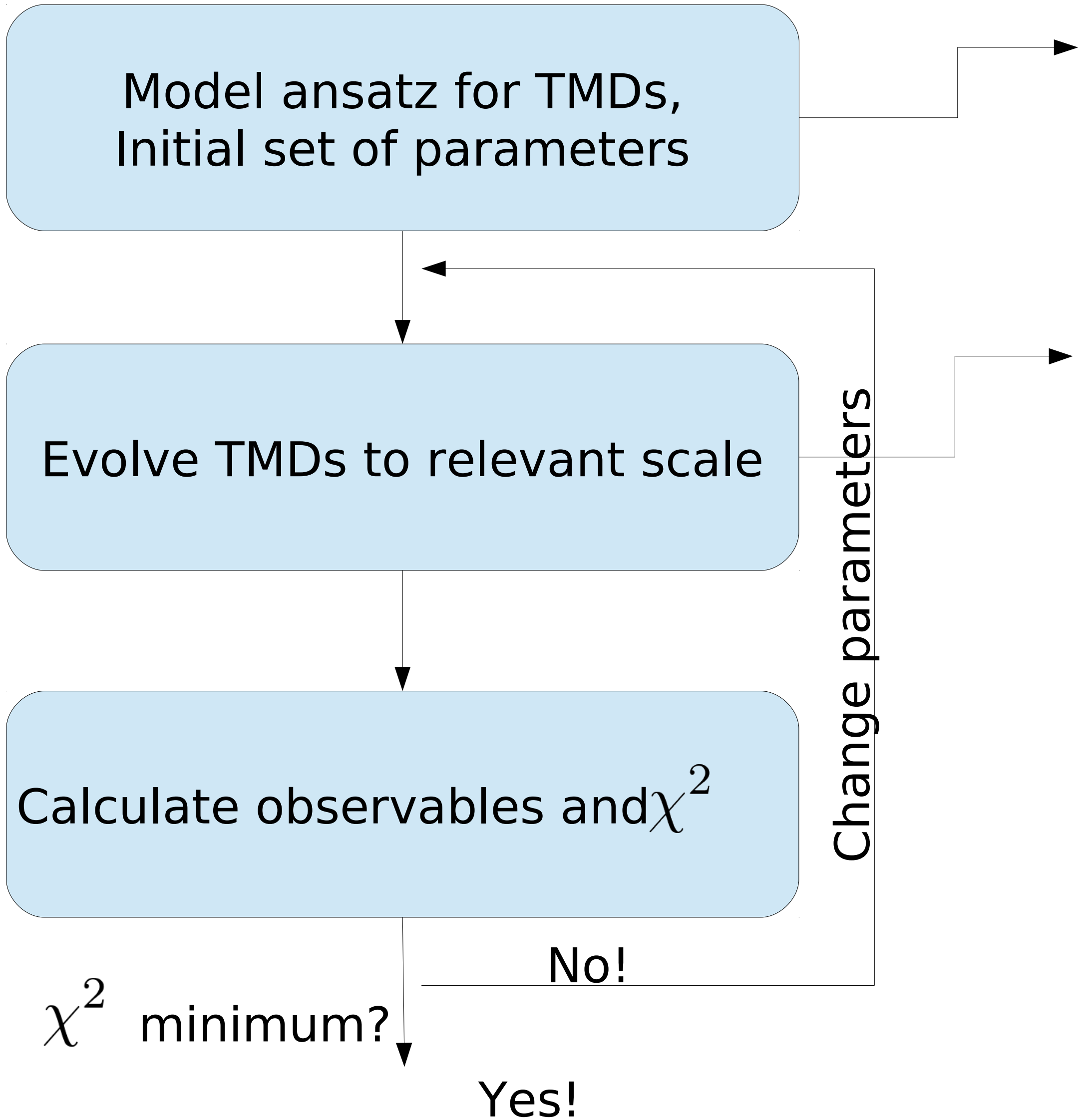
Global analysis:



Parametrization must be flexible but economical in terms of number of parameters

Evolution should be performed fast

Global analysis:

Model ansatz for TMDs, Initial set of parameters

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Calculate observables and χ^2

Number of observables

~1000's to minimize χ^2

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No!

Yes!

Global analysis:

Model ansatz for TMDs,
Initial set of parameters

→ Parametrization must be flexible but economical in terms of number of parameters

Evolve TMDs to relevant scale

→ Evolution should be performed fast

Calculate observables and χ^2

→ Number of observables

~1000's to minimize χ^2

Change parameters

χ^2 minimum?

No!

Yes!

Calculation of TMD expressions is very time consuming!

Why?

Structure functions are convolutions of unobserved momenta:

$$F \sim \int d^2\vec{k}_\perp d^2\vec{p}_\perp \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}) f(x, \vec{k}_\perp) D(z, \vec{p}_\perp)$$

Why?

Structure functions are convolutions of unobserved momenta:

$$F \sim \int d^2\vec{k}_\perp d^2\vec{p}_\perp \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}) f(x,\vec{k}_\perp) D(z,\vec{p}_\perp)$$

Observed in experiment

No analogue of Mellin transform to help to perform this convolution found yet!

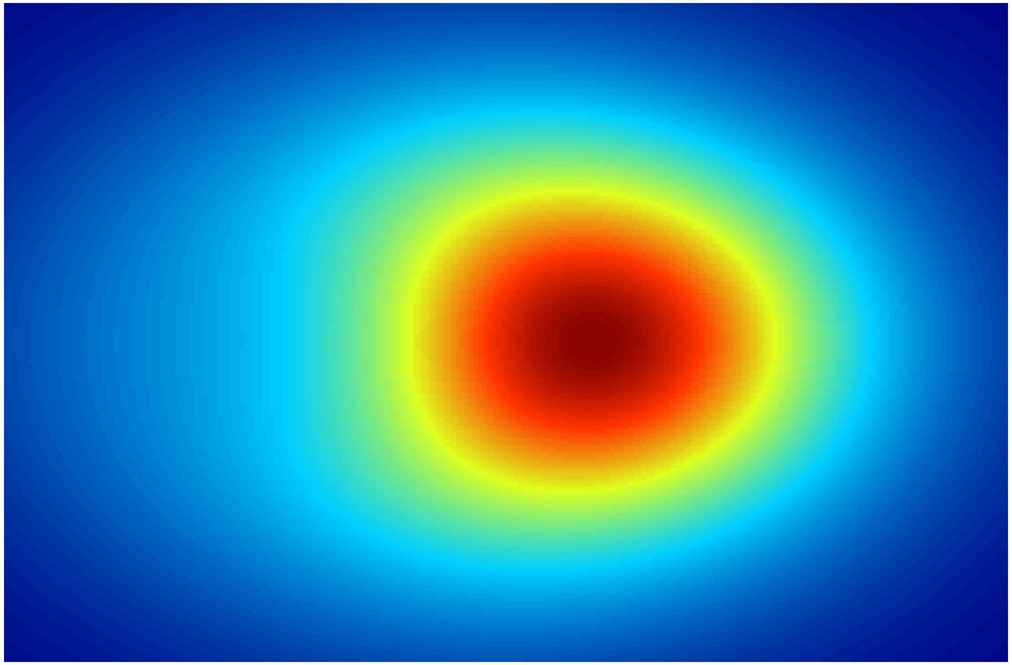
Sivers function

What do we learn from 3D distributions?

$$f(x,\mathbf{k_T},\mathbf{S_T}) = f_1(x,\mathbf{k_T^2}) - f_{1T}^{\perp}(x,\mathbf{k_T^2})\frac{\mathbf{k_{T1}}}{M}$$

The same statement in figures:

This is what we know from experimental data already:



How do we measure Sivers function?

$$A_{UT}^{\sin(\Phi_h-\Phi_S)} = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

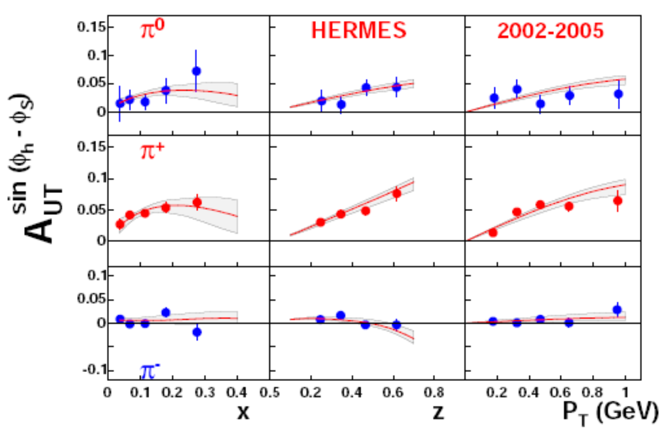
$$\sigma^{\uparrow} - \sigma^{\downarrow} = -f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q} \sin(\phi_h - \phi_S)$$

Unpolarised electron beam
Transversely polarised proton

$$A_{UT}^{\sin(\Phi_h-\Phi_S)} = -\frac{\sum_q e_q^2 f_{1T}^{\perp} \otimes d\hat{\sigma} \otimes D_{h/q}}{\sum_q e_q^2 f_1 \otimes d\hat{\sigma} \otimes D_{h/q}}$$

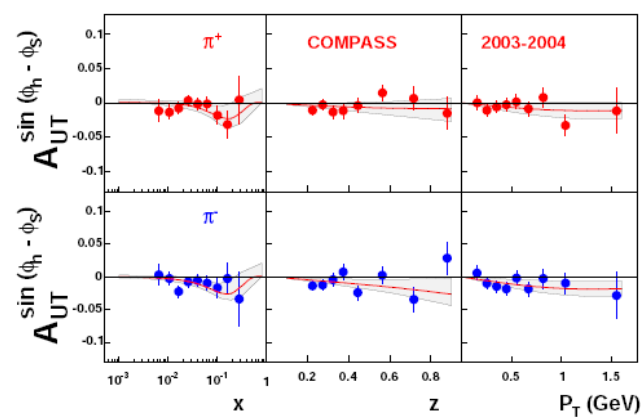
Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel (2006)

HERMES
$ep \to e\pi X$, $p_{lab} = 27.57$ GeV.



Anselmino et al 2010

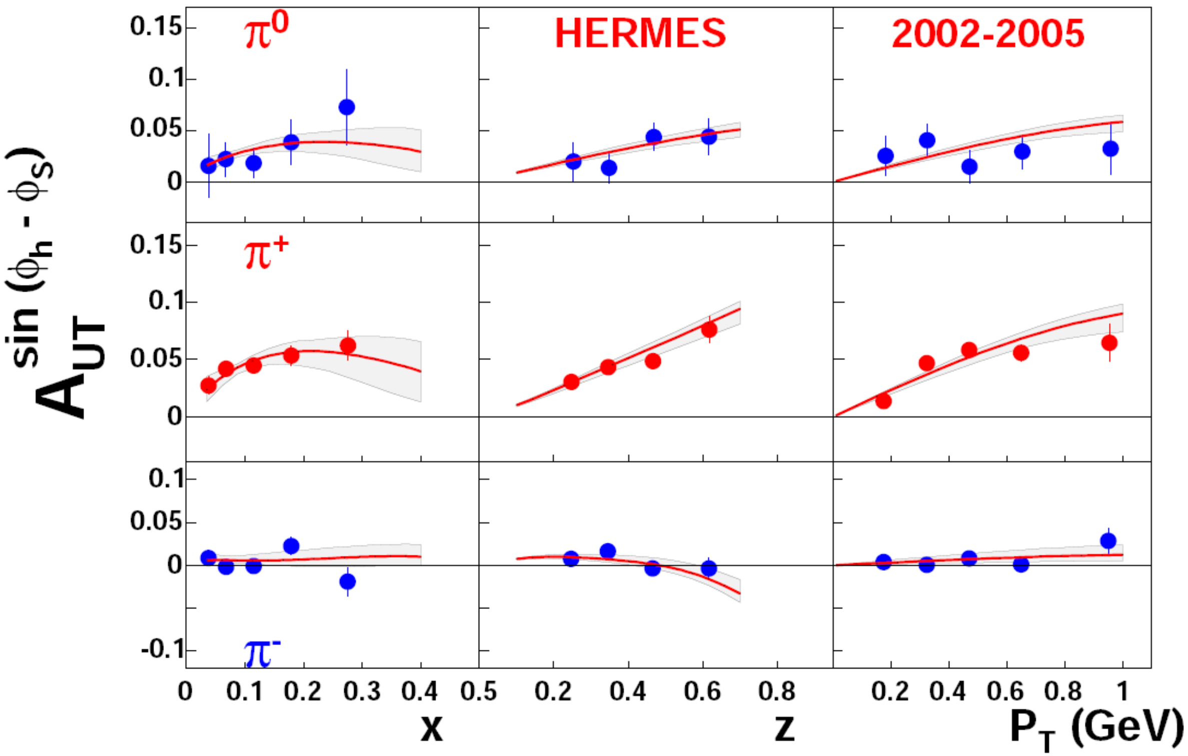
COMPASS
$\mu D \to \mu\pi X$, $p_{lab} = 160$ GeV.



Anselmino et al 2010

Global extractions

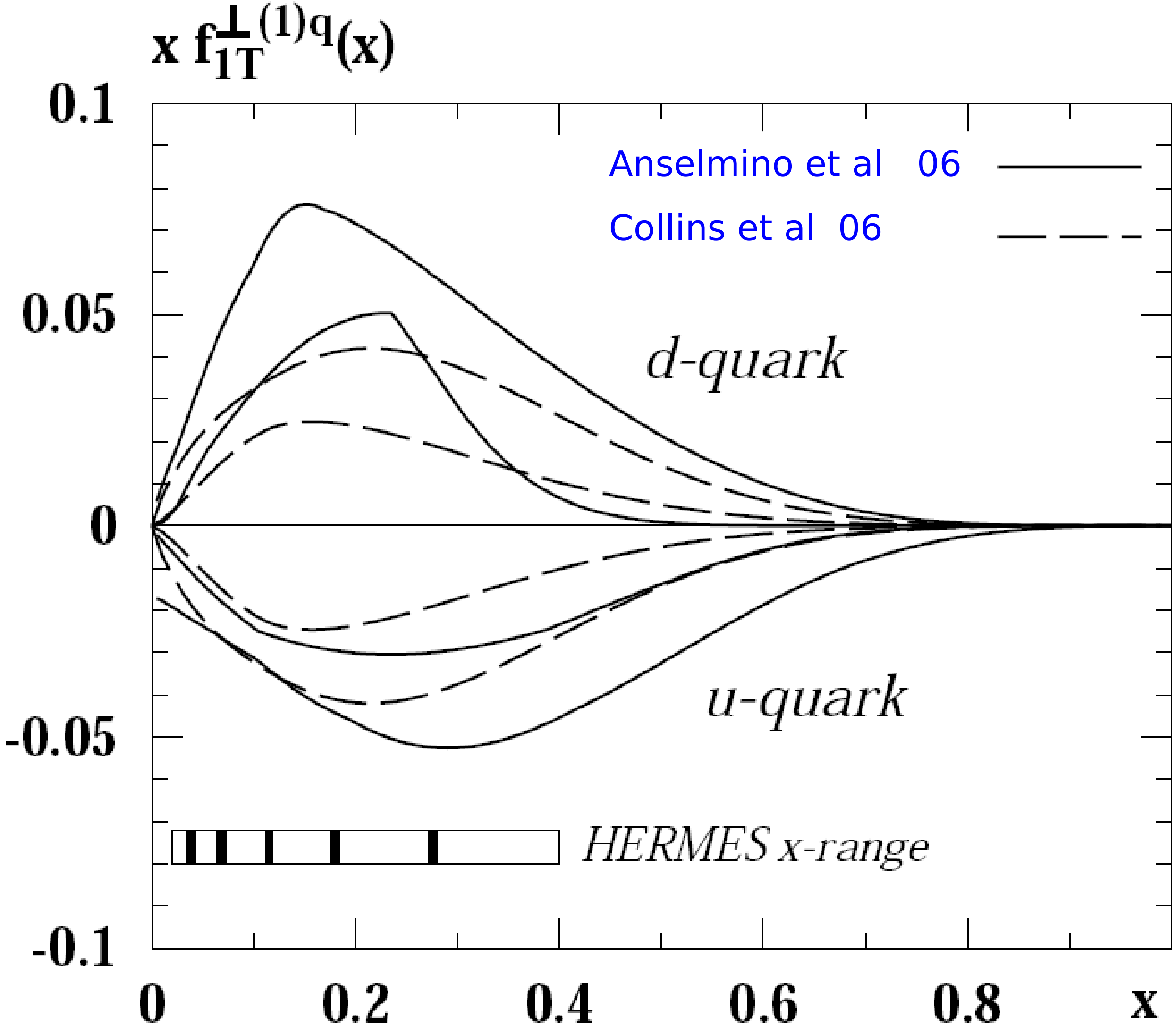
Anselmino et al 2009



HERMES 02 -
COMPASS 04 -
JLAB 11 -

Vogelsang, Yuan 05
Collins et al 06
Anselmino et al 06-09
Bacchetta, Radici 11

Extractions compare well with each other

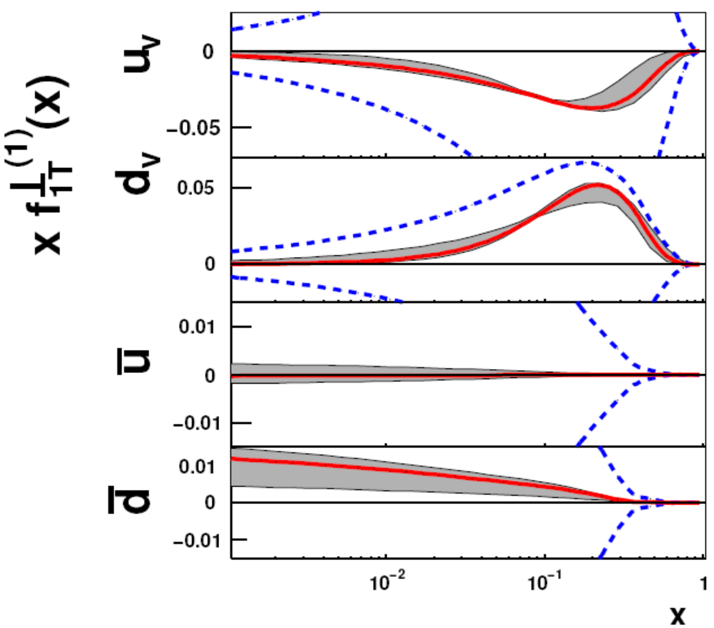


Up and Down Sivers functions have opposite sign

Up quark > 0
Down quark < 0

Extractions compare well with each other

Gamberg, Kang, AP, 13



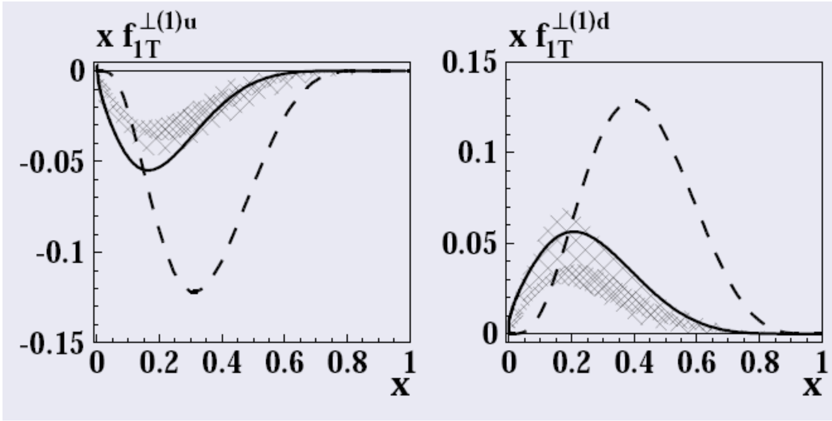
Up and Down
Sivers functions
have opposite sign

Up quark > 0
Down quark < 0

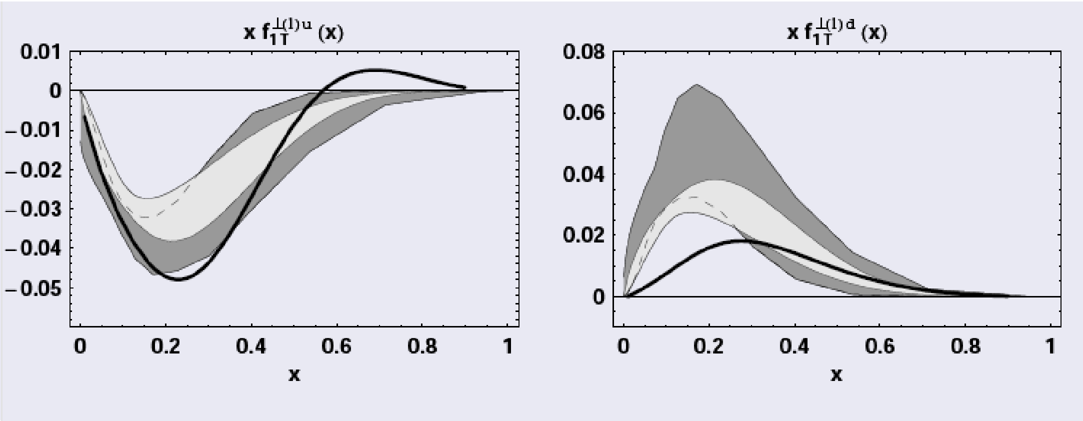
Comparison with models

Light cone wf model Pasquini, Yuan (2011),
Quark-diquark models Bacchetta et al (2010),
Gamberg, Goldstein, Schlegel (2010)
Bag models Yuan (2003), Avakian, Efremov, Schweitzer, Yuan (2010)

Pasquini, Yuan (2011)



Bacchetta et al (2010)



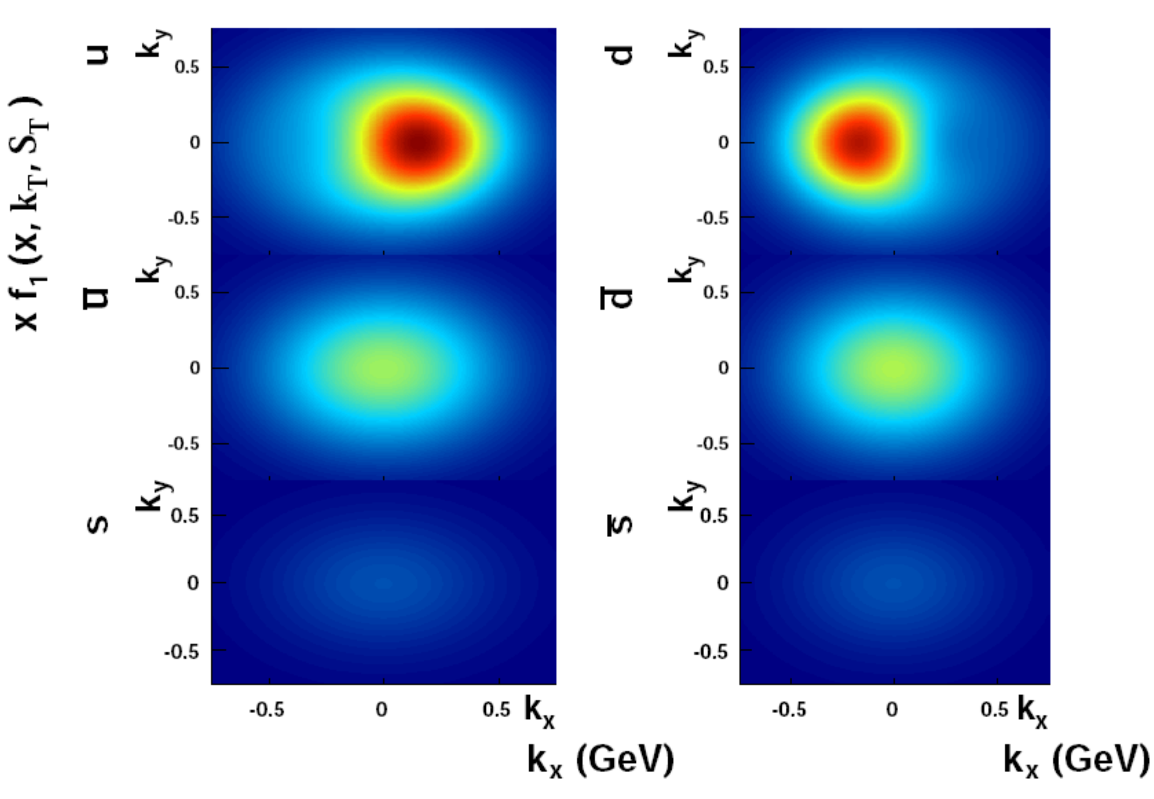
$$f_{1T}^{\perp u} < 0$$

$$f_{1T}^{\perp d} > 0$$

Good agreement.

What do we learn from 3D distributions?

$$f(x,\mathbf{k_T},\mathbf{S_T}) = f_1(x,\mathbf{k_T^2}) - f_{1T}^{\perp}(x,\mathbf{k_T^2})\frac{\mathbf{k_{T1}}}{M}$$



The slice is at:

$$x = 0.1$$

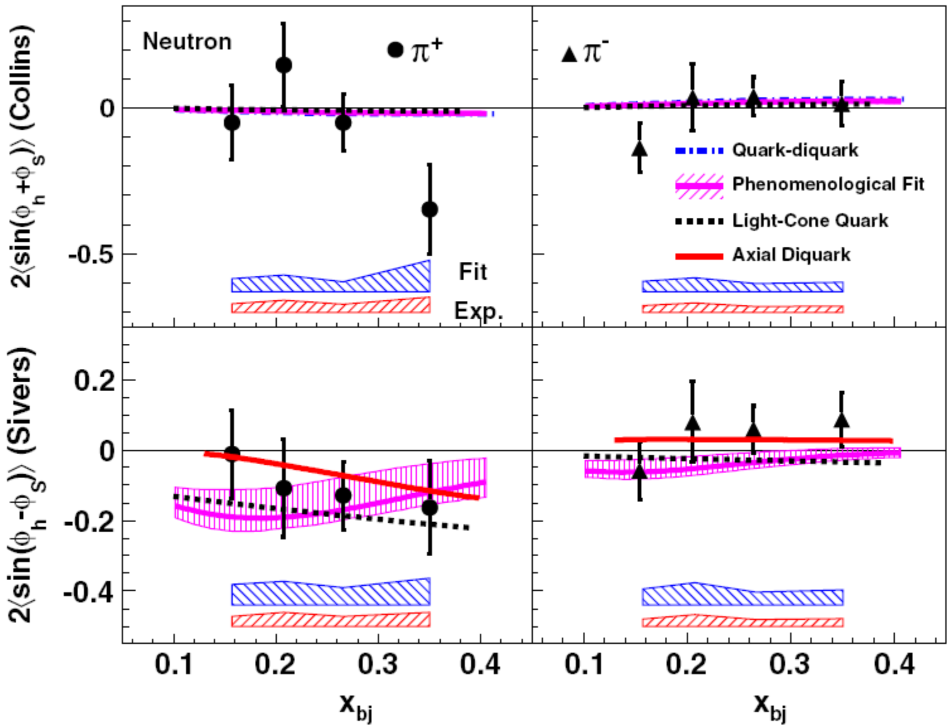
Low-x and high-x region is uncertain
JLab 12 and EIC will contribute

No information on sea quarks

In future we will obtain much clearer picture

Phenomenology

It is extremely important to test our knowledge by **predicting** results of future measurements

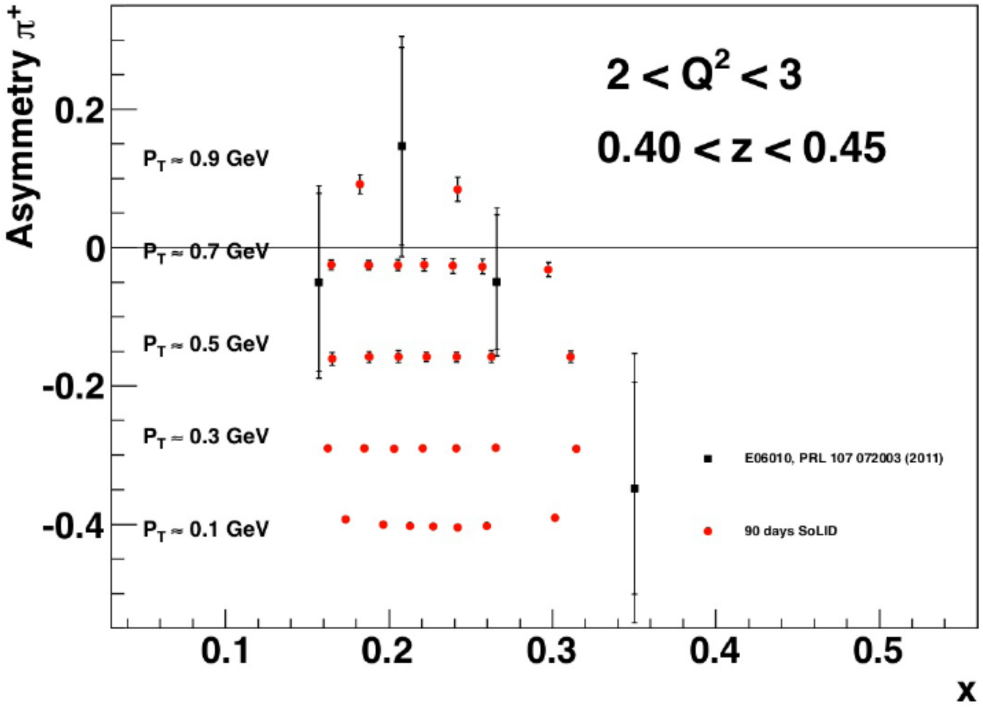


Prediction
Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Melis, **AP**, Turk
EPJA 39 (**2009**) 89-100

Measurement
X. Qian et all (JLab HALL A Coll)
PRL 107 (**2011**) 072003

Perspectives

Projected Data (E12-10-006)



- TMD evolution will be implemented in the fits
- High precision JLab 12 data will test models

Sivers function and twist-3

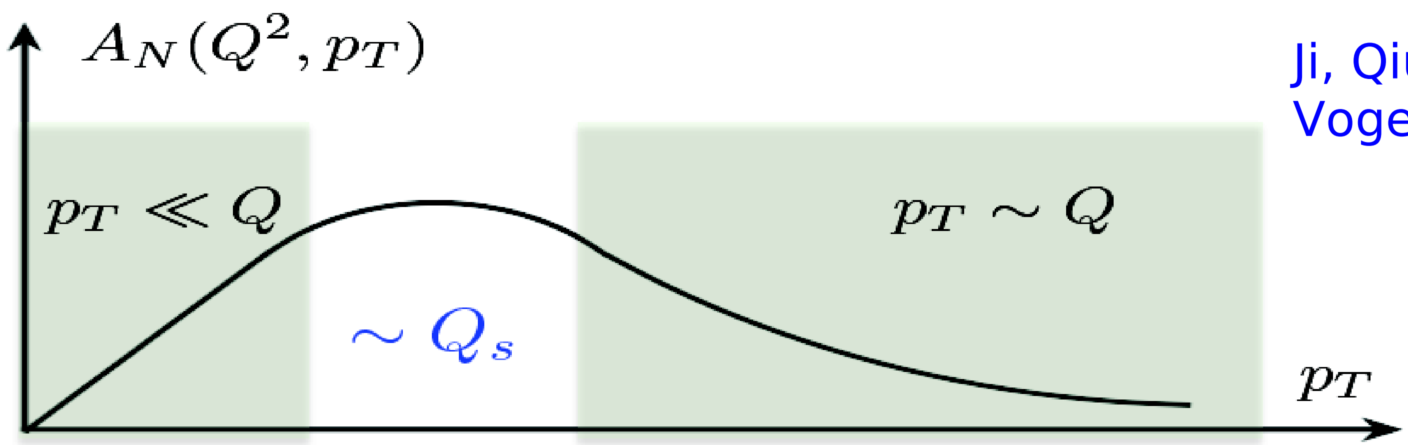
Collinear vs TMD factorization

We can consider two different kinematical regions

$Q_1, Q_2, \ldots \gg \Lambda_{QCD}$ Collinear

$Q_1 \gg Q_2 > \Lambda_{QCD}$ TMD

- Twist-3 – integration over parton momenta
- TMD – direct information on partonic transverse motion



Ji, Qiu, Yuan, Vogelsang, Koike

Consistent in the overlap region!

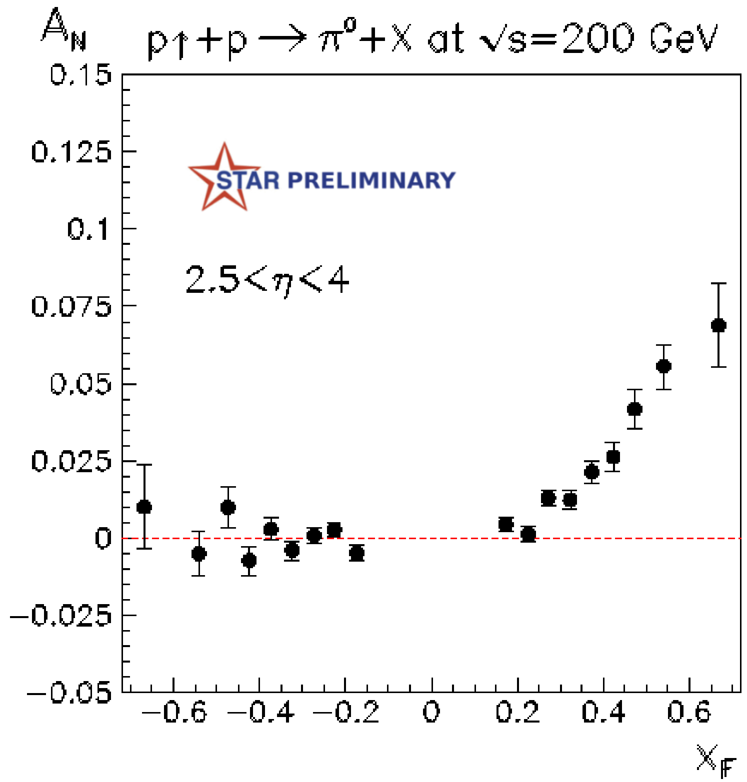
TMDs and twist-3 are related

At operator level:

$$T_F(x,x) = -\int d^2\vec{k}_\perp \frac{k_\perp^2}{M} (f_{1T}^\perp(x,k_\perp))_{SIDIS}$$

Universal in all processes!

Boer,Mulders,Pijlman,
Ji, Qiu,Vogelsang,Yuan,
Koike,Vogelsang,Yuan
Zhou,Yuan,Liang
Bacchetta,Boer,Diehl,Mulders



Asymmetry contains contributions from distribution (Sivers) and fragmentation (Collins)

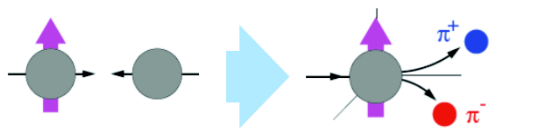
Comparison is difficult:
Sign puzzle

Kang, Qiu, Vogelsang, Yuan (2011)

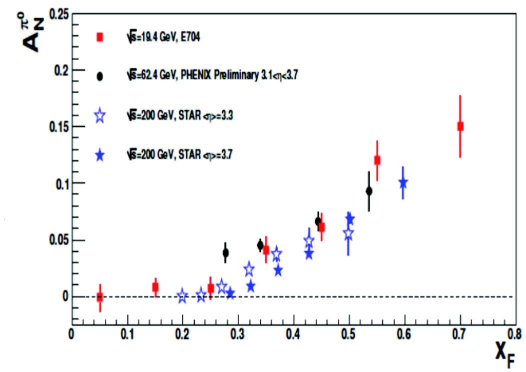
Data analysis

Proton Proton Left -Right asymmetry

$$A(p_A, s_\uparrow) + B(p_B) \to \pi(p) + X$$



$$A(\ell, \vec{s}) \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$



Only one scale P_T

Collinear analysis:

Kouvaris, Qiu,

Vogelsang, Yuan (2006)

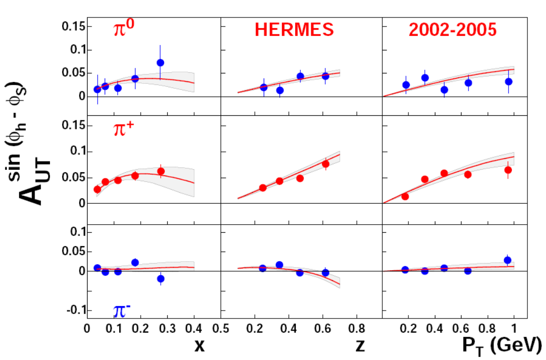
Kanazava, Koike (2010)

TMD analysis:

Anselmino et al (2006)

SIDIS

$$A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \qquad d\sigma^\uparrow - d\sigma^\downarrow \propto \underbrace{f_{1T}^\perp \otimes D_1 \sin(\phi_h - \phi_S)}_{\text{Sivers effect}}$$



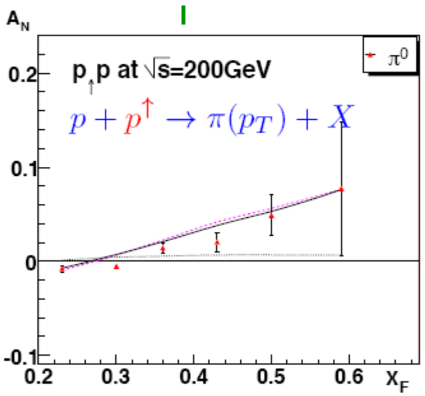
Two scales P_T, Q

TMD analysis: Anselmino et al (2008); Collins et al (2007) ; Vogelsang, Yuan (2006)

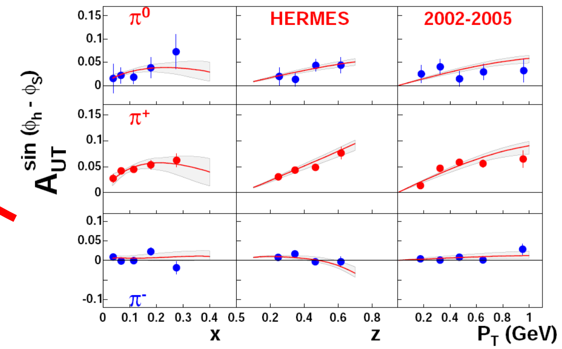
Comparison of results

Kang, Qiu, Vogelsang, Yuan (2011)

$$g_s T_F(x,x) = -2M f_{1T}^{\perp(1)}(x)$$



Collinear analysis: Kouvaris, Qiu, Vogelsang, Yuan (2006)



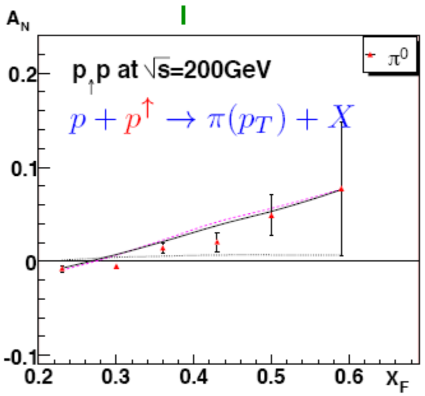
TMD analysis:

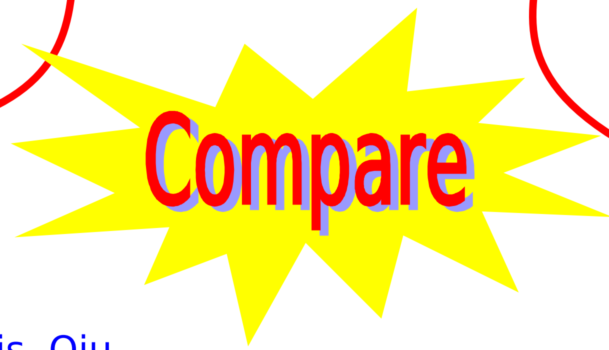
Anselmino et al (2008)

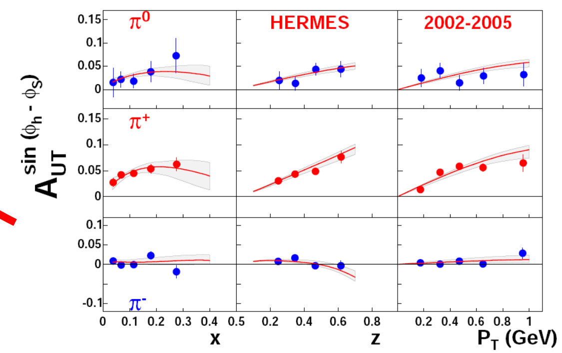
Comparison of results

Kang, Qiu, Vogelsang, Yuan (2011)

$$g_s T_F(x,x) = -2M f_{1T}^{\perp(1)}(x)$$



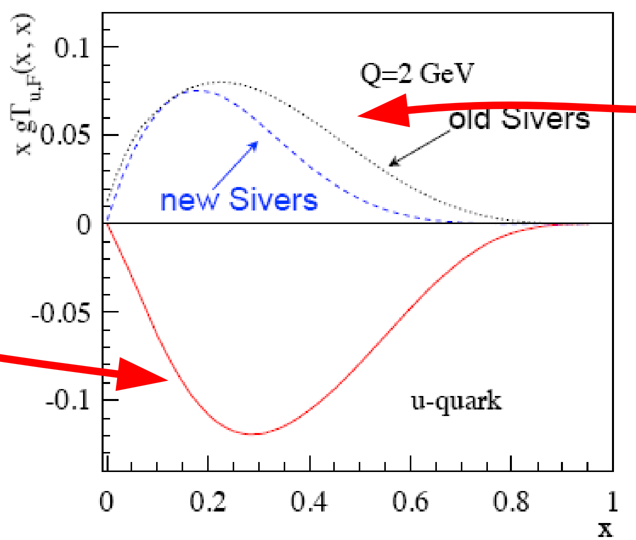




Collinear analysis: Kouvaris, Qiu, Vogelsang, Yuan (2006)

TMD analysis:

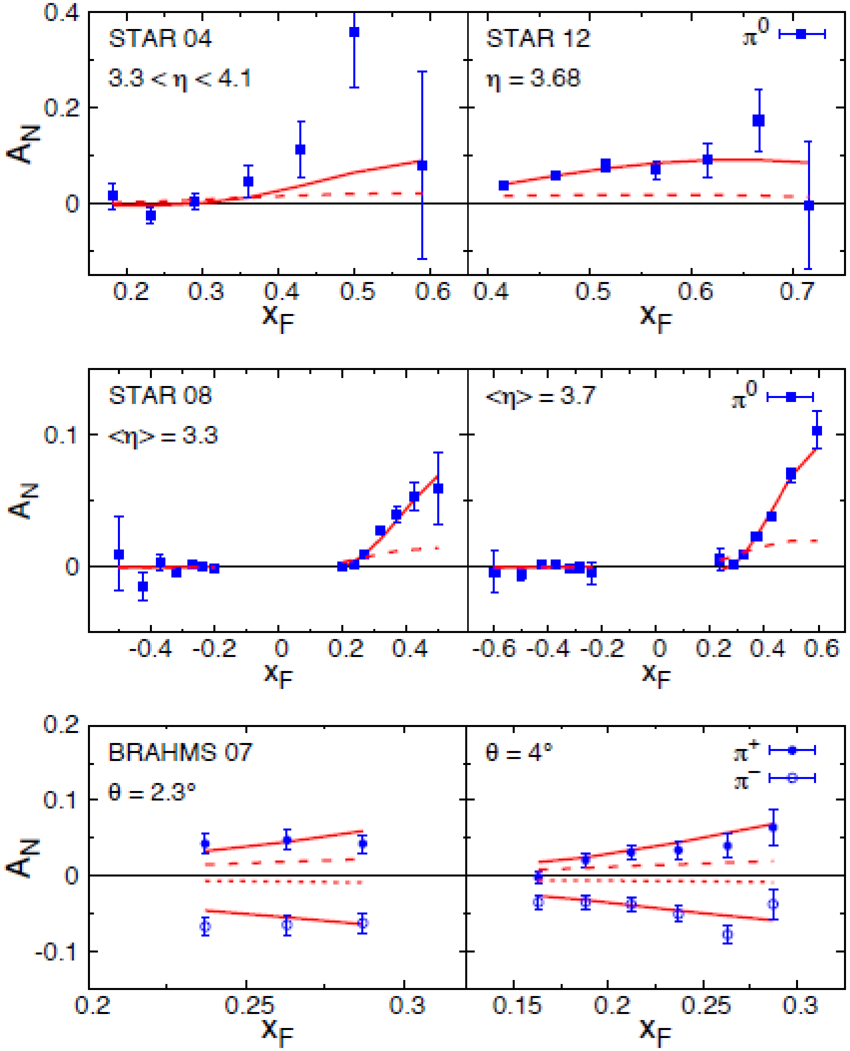
Anselmino et al (2008)



Sign puzzle!

A_N from twist-3 fragmentation functions

(Kanazawa, Koike, Metz, Pitoniak, arXiv:1404.1033)



good fit of A_N mainly due to the new twist-3 fragmentation function

Metz and Pitonyak result

- Calculation of twist-3 fragmentation term (Metz and DP - PLB **723** (2013))

$$\frac{P_h^0 d\sigma_{pol}}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S}\,\epsilon_{\perp\mu\nu}\,S_\perp^\mu P_{h\perp}^\nu \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'}\,\frac{1}{x'S+T/z}\,\frac{1}{-x\hat{u}-x'\hat{t}}$$

$$\times\,\frac{1}{x}\,h_1^a(x)\,f_1^b(x')\left\{\left(\hat{H}^{C/c}(z) - z\frac{d\hat{H}^{C/c}(z)}{dz}\right)S_{\hat{H}}^i + \frac{1}{z}\,H^{C/c}(z)\,S_H^i\right.$$

$$\left. + 2z^2 \int \frac{dz_1}{z_1^2}\,PV\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{C/c,\Im}(z,z_1)\,\frac{1}{\xi}\,S_{\hat{H}_{FU}}^i\right\}$$

➡ "Derivative term" has been calculated previously (Kang, Yuan, Zhou (2010))

➡ First time we have a complete pQCD result for this term in *pp* within the collinear twist-3 approach

$$\hat{H}^{h/q}(z) = z^2 \int d^2\vec{k}_\perp\,\frac{\vec{k}_\perp^{\,2}}{2M_h^2}\,H_1^{\perp h/q}(z, z^2\vec{k}_\perp^{\,2})$$

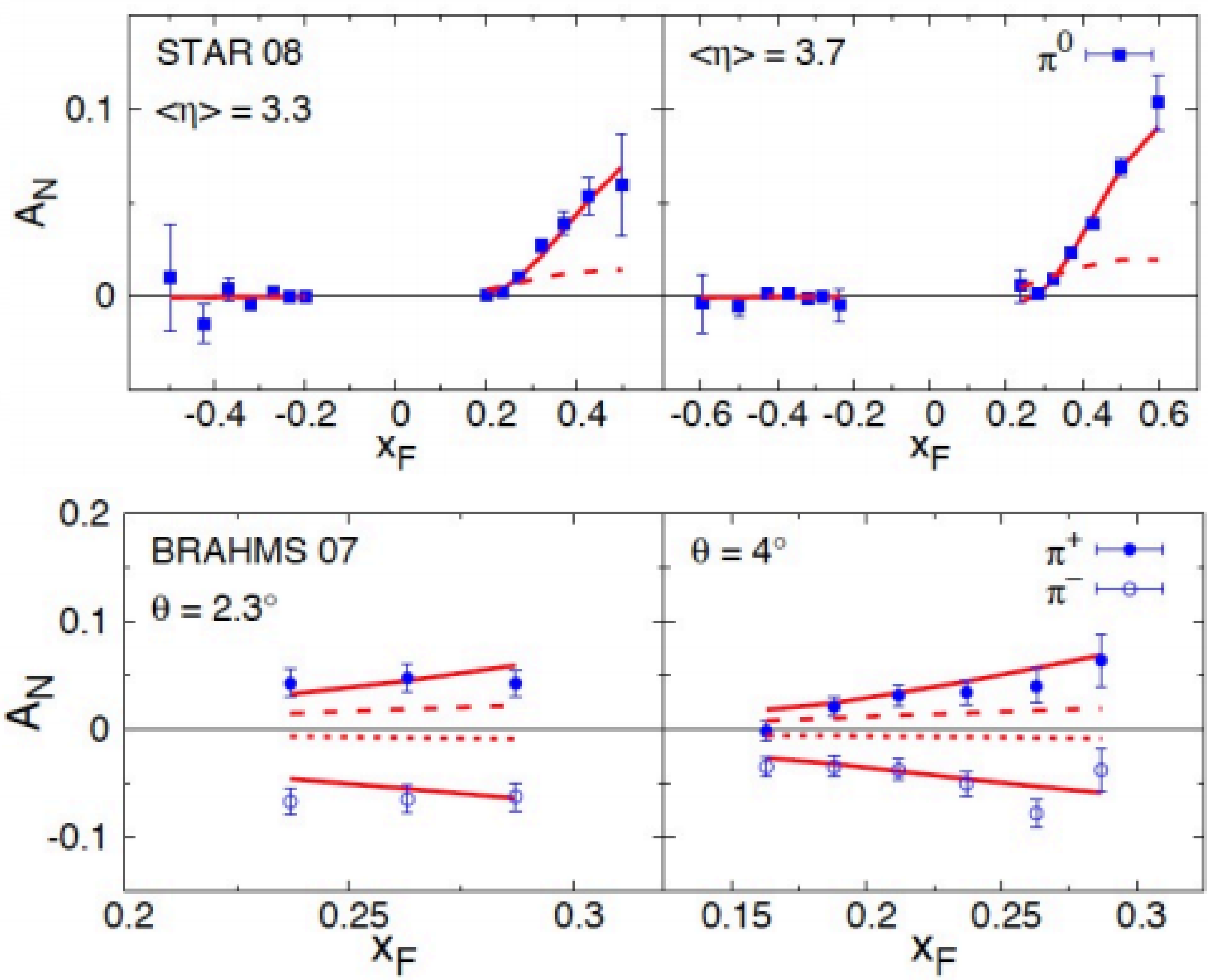
Collins-type function

$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1) = H^{h/q}(z) + 2z\hat{H}^{h/q}(z)$$

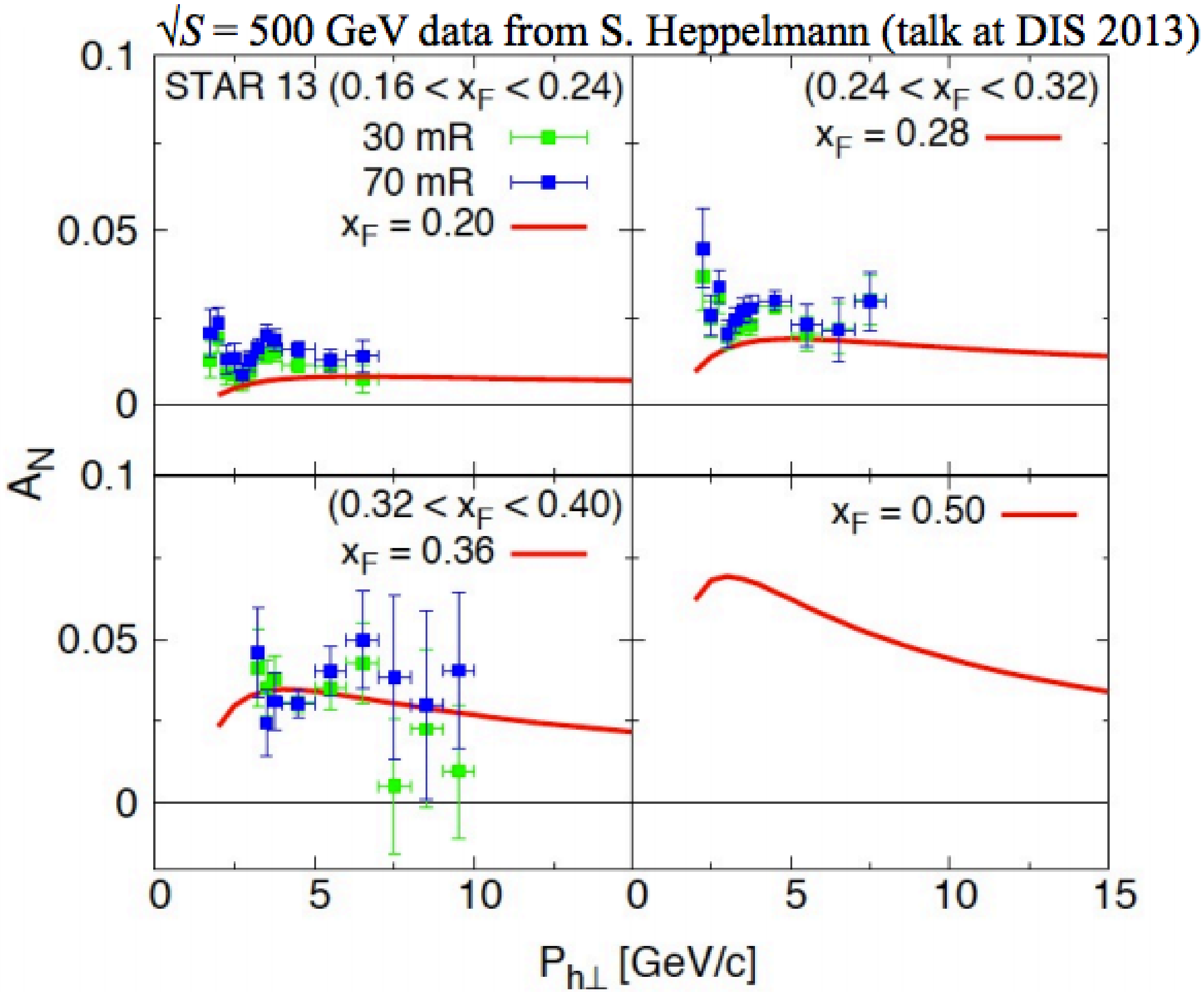
3-parton correlator

Fit the unknown twist-3 FFs

1404.1033



Also pt dependence



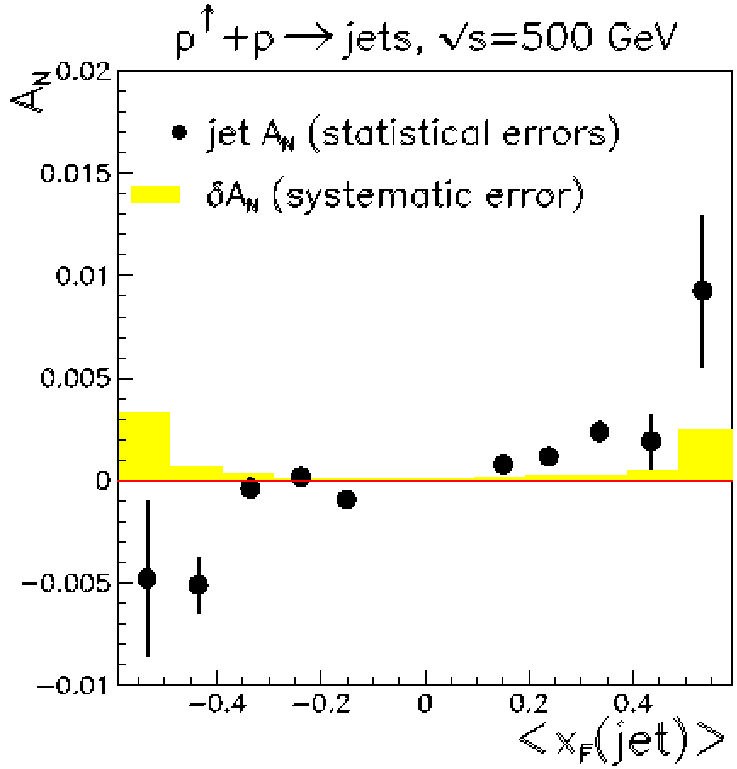
Theory

Note: 500 GeV data was NOT included in the fit

AnDY data on jet AN

Can we measure AN that contains only one of the effects?

Yes! – Jet AN (no fragmentation) has only Sivers like contributions!



$$P^{\uparrow}P \rightarrow JetX$$

AnDY Collaboration (2013)
arXiv:1304.1454

Jet AN contains:
Process dependence → test of the process dependence
Relation twist-3 and TMD → test of twist-3 and TMD relation

Jet AN

We calculate jet AN in twist-3:

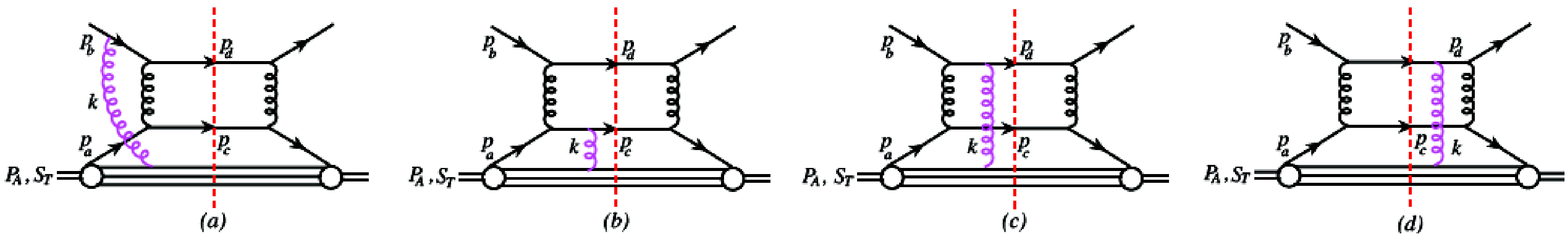
$$E_J \frac{d\Delta\sigma(s_\perp)}{d^3 P_J} = \epsilon_{\alpha\beta} s_\perp^\alpha P_{J\perp}^\beta \frac{\alpha_s^2}{s} \sum_{a,b} \int \frac{dx}{x} \frac{dx'}{x'} f_{b/B}(x') \times \left[T_{a,F}(x,x) - x \frac{d}{dx} T_{a,F}(x,x) \right] \times \frac{1}{\hat{u}} H_{ab\to c}^{\mathrm{Sivers}}(\hat{s},\hat{t},\hat{u}) \delta\left(\hat{s}+\hat{t}+\hat{u}\right),$$

Process dependence is here

Gamberg, Kang, AP (2013)

Jet AN

We calculate jet AN in twist-3: Gamberg, Kang, (2011)



Both initial and final state interactions contribute

$$f_{1T}^{\perp a, qq' \to qq'} = -\frac{3}{N_c^2 - 1} f_{1T}^{\perp a, SIDIS}$$

Process dependence is here

Many other partonic channels $qg \to qg,\ \bar{q}q \to gg \ldots$

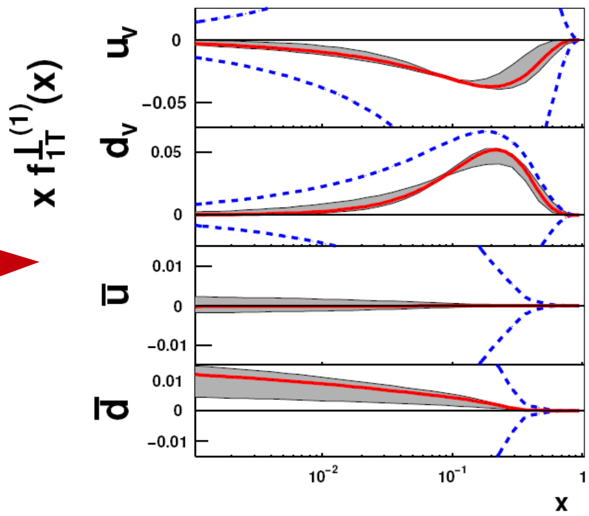
Jet AN

We calculate jet AN in twist-3:

$$E_J \frac{d\Delta\sigma(s_\perp)}{d^3P_J} = \epsilon_{\alpha\beta} s_\perp^\alpha P_{J\perp}^\beta \frac{\alpha_s^2}{s} \sum_{a,b} \int \frac{dx}{x} \frac{dx'}{x'} f_{b/B}(x')$$
$$\times \left[T_{a,F}(x,x) - x \frac{d}{dx} T_{a,F}(x,x) \right]$$
$$\times \frac{1}{\hat{u}} H^{\rm Sivers}_{ab\to c}(\hat{s},\hat{t},\hat{u}) \delta\left(\hat{s}+\hat{t}+\hat{u}\right),$$

Twist-3 TMD relation

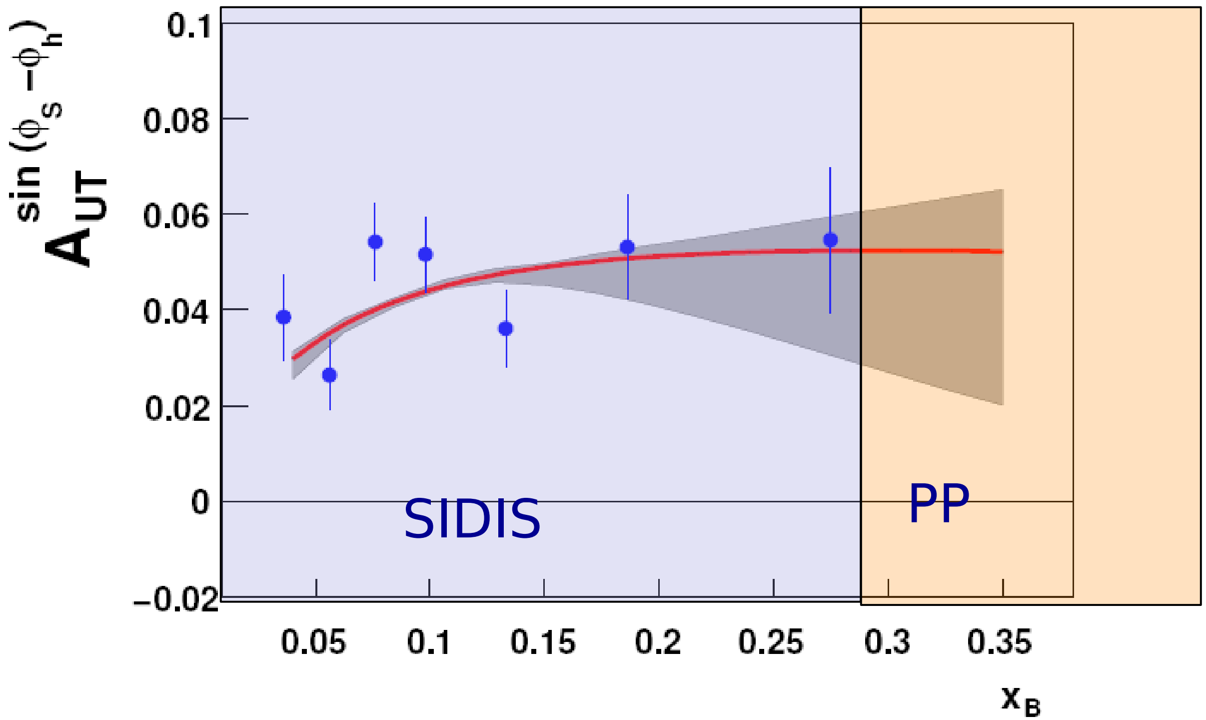
Use Sivers that describes SIDIS:



Gamberg, Kang, AP (2013)

Jet AN

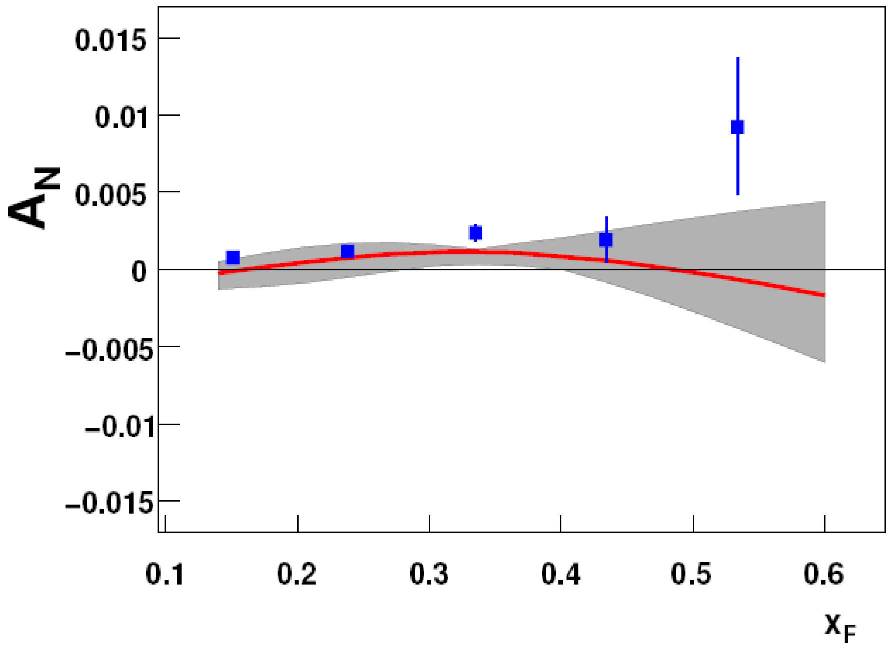
Jet AN corresponds to high x region which is not yet accessible in SIDIS → refit SIDIS data in order to explore high x region



Gamberg, Kang, AP (2013) compatible with Anselmino et al (2009)

Jet AN

Compare with AnDY data:

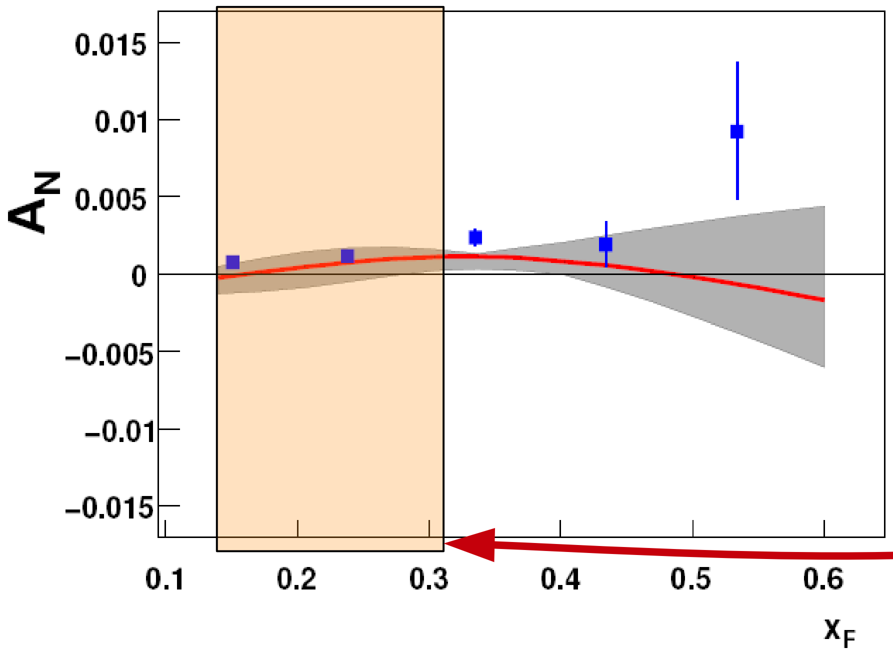


$$\langle y \rangle = 3.25, \ \sqrt{s} = 500(GeV)$$

Gamberg, Kang, AP (2013)

Jet AN

Compare with AnDY data:



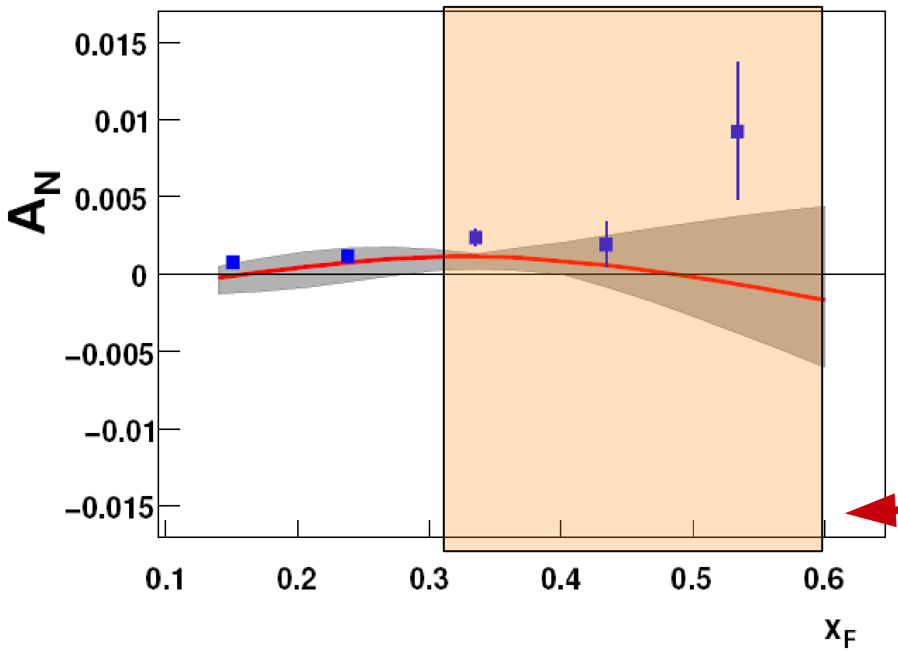
This region corresponds to SIDIS kinematical region: agreement is very encouraging

$$\langle y \rangle = 3.25, \ \sqrt{s} = 500(GeV)$$

Gamberg, Kang, AP (2013)

Jet AN

Compare with AnDY data:



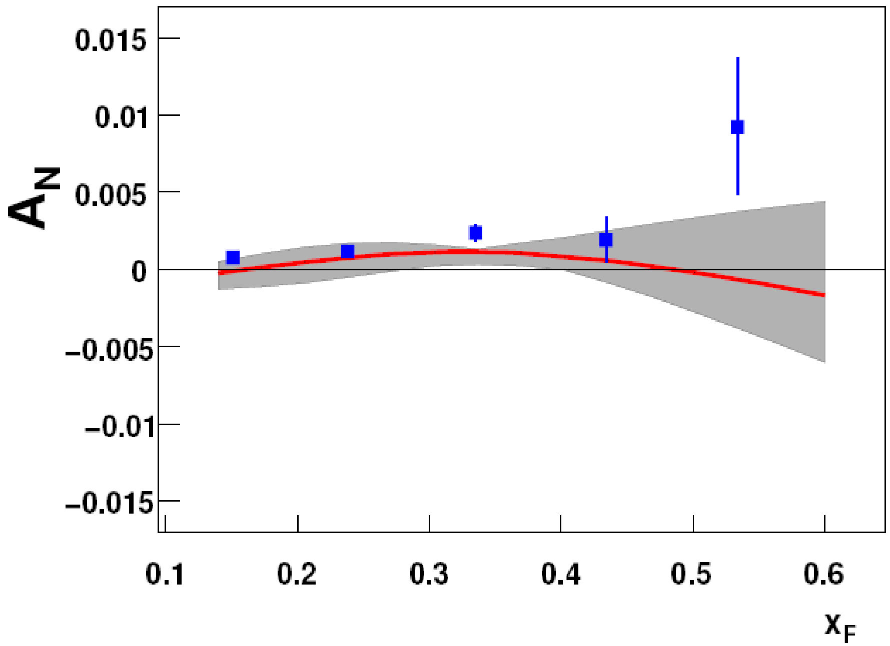
This region relies on large-x region, future JLab 12 measurement is important

$$\langle y \rangle = 3.25, \ \sqrt{s} = 500(GeV)$$

Gamberg, Kang, AP (2013)

Jet AN

Compare with AnDY data:



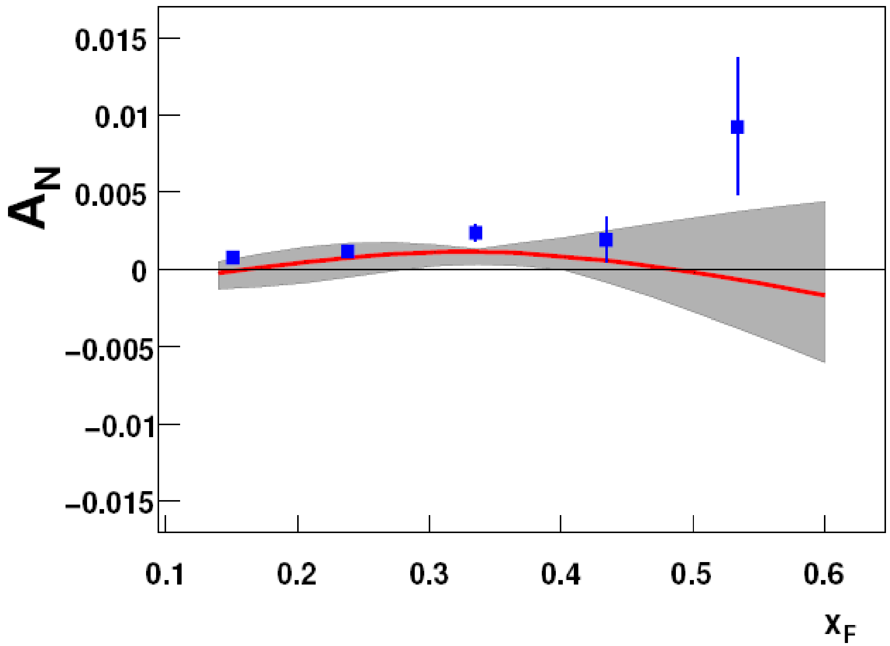
- ✔ The sign is correct
- ✔ The size is correct

$$\langle y \rangle = 3.25, \ \sqrt{s} = 500(GeV)$$

Gamberg, Kang, AP (2013)

Jet AN

Compare with AnDY data:



$$\langle y \rangle = 3.25, \ \sqrt{s} = 500(GeV)$$

Gamberg, Kang, AP (2013)

- ✔ The sign is correct
- ✔ The size is correct

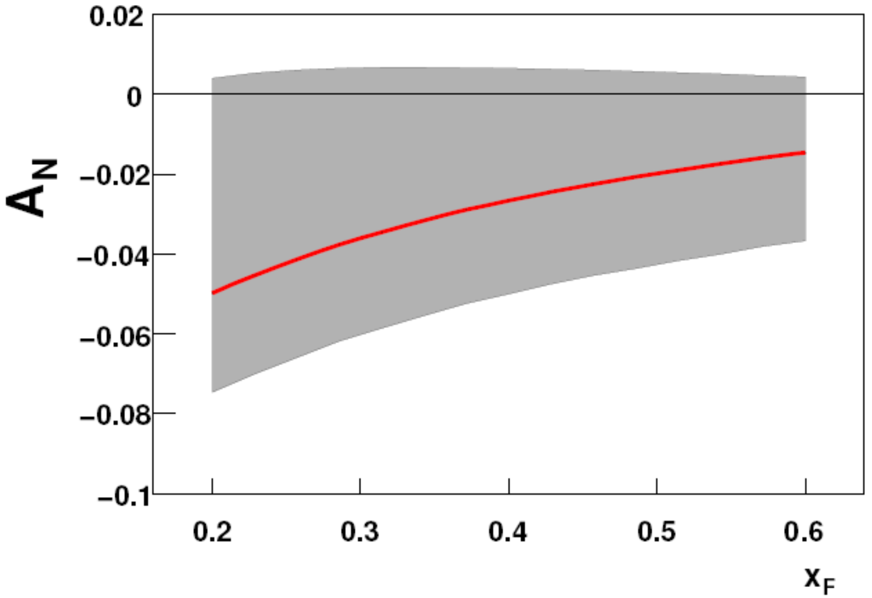
Result is indication

- ✔ TMD and twist-3 are compatible
- ✔ Sivers effect is process dependent

Fundamental tests of QCD!

Future

Direct photon production $P^{\uparrow}P \to \gamma X$



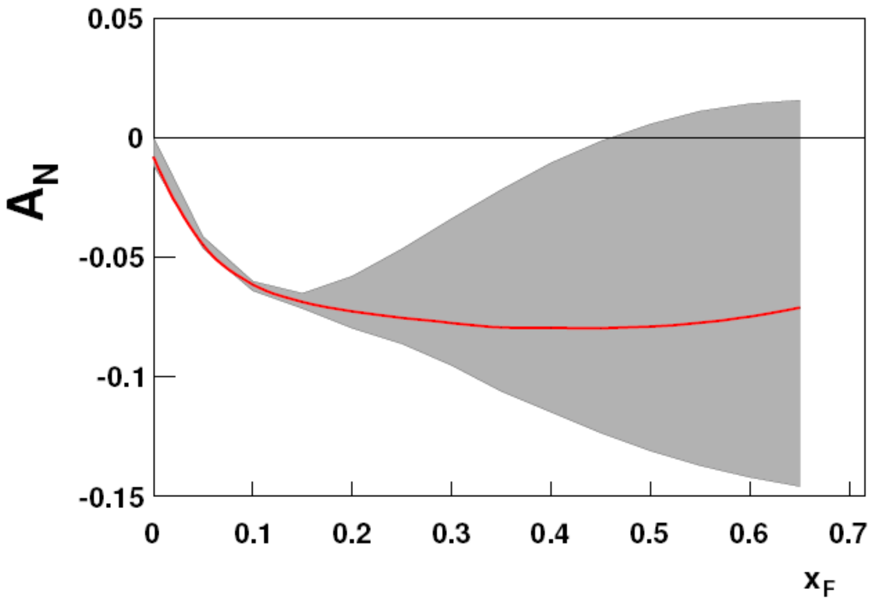
- Bigger asymmetry
- This measurement allows to test consistency of TMD and twist-3 factorizations

$$\langle y \rangle = 3.5,\ \sqrt{s} = 200(GeV)$$

Gamberg, Kang, AP (2013)

Future

Drell-Yan $P^{\uparrow}P \to \ell^{+}\ell^{-}X$



- This measurement proves directly process dependence of Sivers effect

$$4 < Q < 8(GeV) \; \sqrt{s} = 500(GeV)$$

Gamberg, Kang, AP (2013)